Optimal Monetary and Fiscal Policy in a Liquidity Trap *

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Recent developments in both Japan and the U.S. have brought new attention to the question of how policy should be conducted when short-term nominal interest rates reach a level below which no further interest-rate declines are possible (as in Japan), or below which further interest-rate declines are regarded as undesirable (arguably the situation of the U.S.). It is sometimes feared that when nominal interest rates reach their theoretical or practical lower bound, monetary policy will become completely impotent to prevent either persistent deflation or persistent underutilization of productive capacity. The experience of Japan for the last several years suggests that the threat is a real one.

In a previous paper (Eggertsson and Woodford, 2003), we consider how the existence of a theoretical lower bound for nominal interest rates at zero affects the optimal conduct of monetary policy, under circumstances where the natural rate of interest — the real interest rate required for an optimal level of utilization of existing productive capacity — can be temporarily negative, as in Krugman's (1998) diagnosis of the recent situation in Japan. We show that the zero lower bound can be a significant obstacle to macroeconomic stabilization at such a time, through an approach to the conduct of monetary policy that would be effective under more normal circumstances. Nonetheless, we find that the distortions created by the zero lower bound can be mitigated to a large extent, in principle, through commitment to the right kind of policy. We show that an optimal policy is *history-dependent,* remaining looser after the real disturbance has dissipated than would otherwise be chosen given the conditions prevailing at that time. According to our model, the expectation that interest rates will be kept low for a time even after the natural rate of interest has returned to a positive level can largely eliminate the deflationary and contractionary impact of the disturbance that temporarily causes the natural rate of interest to be negative.

An important limitation of our previous analysis is that it abstracted entirely from the role of fiscal policy in coping with a situation of the kind that may give rise to a liquidity trap. In the model of Eggertsson and Woodford (2003), fiscal policy considerations of two distinct sorts are omitted. First, in the consideration of optimal policy there, no fiscal instruments are assumed to be available to the policymaker. The sole problem considered was the optimal conduct of monetary policy, taking fiscal policy as given, and assuming that fiscal policy fails to eliminate the temporary decline in the natural rate of interest that created a challenge for monetary policy. And second, the fiscal consequences of alternative monetary policies are ignored in the characterization of optimal monetary policy. It is thus implicitly assumed (as in much of the literature on the evaluation of alternative monetary policy rules) that the distortions associated with an increase in the government's revenue needs are of negligible importance relative to the distortions resulting from the failure of prices to adjust more rapidly when considering alternative monetary policies. This would be literally correct if a lump-sum tax were available as a source of revenue, but is not correct (at least, not completely correct) given that only distorting taxes exist in practice.

In the present paper, we seek to remedy both omissions by extending our analysis to take account of the consequences of tax distortions for aggregate economic activity and pricing decisions. The model that we consider introduces a distorting tax (which we model as a VAT) that is assumed to be the only available source of government revenues, and considers both the optimal conduct of monetary policy (in particular, the optimal evolution of short-term nominal interest rates) and the optimal timing of tax collections in such a setting. Our key result is an extension of the analysis of Benigno and Woodford (2003) to a case in which the zero lower bound on nominal interest rates is a binding constraint on what can be done with monetary policy.

There are several important issues that we wish to clarify with such an investigation. First, we wish to understand the implications of an occasionally binding zero lower bound for optimal tax policy. Feldstein (2002) has suggested that while tax policy is not a useful instrument of stabilization policy under normal circumstances — this problem being both adequately and more efficiently addressed by monetary policy, because of the greater speed and precision with which central banks can respond to sudden economic developments — there may nonetheless be an important role for fiscal stabilization policy when a binding zero bound constrains what can be done through monetary policy. Here we consider this issue by analyzing optimal fiscal policy in a setting that has been contrived to yield the result that it is not optimal to vary the tax rate in response to real disturbances, as long as these do not cause the zero bound to bind.[1] We find that it is indeed true that an optimal tax policy involves changing tax rates in response to a situation in which the zero bound is temporarily binding, and we find furthermore that the optimal change in tax rates is largely temporary. However, the nature of the optimal tax response to a liquidity trap is quite different from traditional Keynesian policy advice. In the case that only taxes with supply-side effects are available, we find that it is actually optimal to *raise* taxes while the economy is in a liquidity trap. And while we find that Feldstein is correct to argue that tax policy can eliminate the problem of the zero bound in principle, the conditions under which this can be done are somewhat more special than his discussion suggests.

Second, we wish to consider the robustness of our previous conclusions, about the importance of commitment by the central bank to a history-dependent monetary policy following a period in which the zero bound binds, to allowing for the use of fiscal policy in a way that mitigates the effects of the real disturbance to the extent possible. Many readers have worried that the demonstration in Eggertsson and Woodford (2003) of a dramatic benefit from commitment to a history-dependent monetary policy depends on having excluded from consideration the traditional Keynesian remedy for a liquidity trap, *i.e.,* countercyclical fiscal policy.[2] Not only might monetary policy be unimportant once a vigorous fiscal response is allowed, but commitment of future policy and signalling of such commitments might also be found to be of minor importance, once one introduces an instrument of policy (tax incentives) that can affect spending and pricing decisions quite independently of any change in expectations regarding future policy.

We address this issue by considering optimal monetary policy when tax policy is used

[1] The real reason that Feldstein does not consider tax policy to be a useful tool of everyday stabilization policy, of course, is not the one on which our model relies; for we assume, for purposes of normative analysis, that tax rates can be quickly adjusted on the basis of full information about current aggregate conditions. But by considering a case in which it would be optimal to fully smooth tax rates at times when the zero bound does not bind (and has not recently bound), we can show clearly that the zero bound introduces a new reason for it to be desirable for tax rates to depend on aggregate conditions.

[2] At the Brookings panel meeting at which our previous paper was presented, a number of panel members protested the omission of any role for fiscal policy; see the published discussion of Eggertsson and Woodford (2003).

optimally, and examining the degree to which it involves a commitment to history-dependent policy. We find that except under the most favorable circumstances for the effective use of fiscal policy, optimal monetary policy continues to be similar in important respects to the optimal policy identified in our previous paper. In particular, it requires the central bank to commit itself to maintain a looser policy *following* a period in which the natural rate of interest has been negative (and the zero interest-rate bound has been reached) than would otherwise be optimal given conditions at the later time. This implies a temporary period of inflation and eventual stabilization of the price level around a higher level than would have been reached if the zero bound had not been hit. On the other hand, the welfare gains from such a sophisticated monetary policy commitment are relatively modest when fiscal policy is used for stabilization purposes, and if fiscal policy is sufficiently flexible, they disappear altogether.

We further show that when the set of available tax instruments is restricted in a way that seems to us fairly realistic, it is also optimal for tax policy to be conducted in a history-dependent way. The policy authorities should commit to a more expansionary fiscal policy (*i.e.,* lower tax rates) after the disturbance to the natural rate of interest has ended; thus there should be a commitment to use both monetary and fiscal policy to create "boom" conditions at that time. Whether monetary policy is optimal or not, the optimal fiscal policy is history-dependent and depends on successful advance signalling of policy commitments.

In fact, we compare the outcome with fully optimal monetary and fiscal policies with the best outcome that can be achieved by purely forward-looking policies. We find that the choice of an optimal fiscal rule, even subject to the restriction that policy be purely forward-looking (in the sense of Woodford, 2000), allows substantial improvement over the outcome that would result from a forward-looking monetary policy (*i.e.,* a constant inflation target) in the case of a simple tax-smoothing rule for taxes. Nonetheless, further improvements in stabilization are possible through commitment to history-dependent policies.

It is also worth noting that the gains from fiscal stabilization policy that we find, even when policy is constrained to be purely forward-looking, are always heavily dependent on the public's correct understanding of how current developments change the outlook for *future* policy. Optimal fiscal policy involves raising taxes during the liquidity trap in order to lower the public debt (or build up government assets), implying that taxes will be lower later. The expectation of lower taxes later can be created even under the constraint that fiscal policy be purely forward-looking, because the level of the public debt is a state variable that should condition future tax policy even when policy is purely forward-looking. But the effectiveness of the policy does depend on the public's expectations regarding future policy changing in an appropriate way when the disturbance occurs, and so there remains an important role for discussion by policymakers of the outlook for future policy.

Finally, we wish to re-examine the character of optimal monetary policy taking account of the fiscal effects of monetary expansion. Auerbach and Obstfeld (2003) emphasize that when tax distortions are considered, there is an additional benefit from expansionary monetary policy in a liquidity trap, resulting from reduction of the future level of real tax collections that will be needed to service the public debt. We consider this issue by analyzing optimal monetary policy in a setting where only distorting sources of government revenues exist, and where there is assumed to be an initial (nominal) public debt of non-trivial magnitude.

An important issue that is not considered in Auerbach and Obstfeld's calculation of the welfare gains from monetary expansion is the extent to which the gains that they find would also be present even if the economy were *not* in a liquidity trap — and thus constitute an argument, not for unusual monetary expansion in the event of a liquidity trap, but for *always* expanding the money supply. Of course, as is well known from the literature on rules versus discretion, it is easy to give reasons why monetary expansion should appear attractive to a discretionary policy authority, that asks, at a given point in time, what the best equilibrium would be from that time onward, taking as given past expectations and not regarding itself as bound by any past commitments. At the same time, in several well-known models, the authority ought to prefer to commit itself in advance *not* to behave this way, owing to the harmful consequences of prior anticipation of inflationary policy. It is important to consider the extent to which the gains from expansionary monetary policy under circumstances of a liquidity trap are ones that one would commit oneself in advance to pursue under such a contingency, or whether they represent the sort of temptation under discretionary policy that a sound policy must commit itself to resist.

We address this question by considering optimal state-contingent policy under advance commitment. While we do consider how policy should be conducted from some initial date at which it is already known that a disturbance that lowers the natural rate of interest has occurred, we consider this question from a "timeless perspective," as advocated by Woodford (1999; 2003, chap. 7); this means that characterize a policy from that date forward to which the policy maker would have wished to commit itself at an earlier date.[3] We find that such a commitment will involve a zero inflation rate over periods when the zero bound does not bind and has not recently bound; but that the central bank will commit itself to a policy that permanently increases the price level by a finite proportion each time the zero bound is reached. We also compare the size of the optimal increase in the price level, for a given size and duration of real disturbance, for high-debt versus low-debt economies, in order to see to what extent the existence of a higher shadow value of additional government revenues (in order to reduce tax distortions) strengthens the case for a commitment to expansionary policy under circumstances of a liquidity trap. We find that optimal policy is somewhat more inflationary (in response to a real disturbance that lowers the natural rate of interest) in the case of an economy with a larger quantity of nominal public debt and more severe tax distortions; however, neither optimal fiscal policy nor optimal monetary policy are fundamentally different in this case than they are in the simpler case of an economy with zero initial public debt and a zero steady-state tax rate.

1 An Optimizing Model with Tax Distortions

The framework that we use to analyze the questions posed above is the one introduced in Benigno and Woodford (2003). We first review the structure of the model, and then

[3]See also Svensson and Woodford (2004), Giannoni and Woodford (2002), and Benigno and Woodford (2003), for further discussions of this concept.

the linear-quadratic approximation derived by Benigno and Woodford. We finally discuss the additional complications that are introduced in the case that the zero lower bound on nominal interest rates is sometimes a binding constraint on monetary policy.

1.1 The Exact Policy Problem

Here we review the structure of the model of Benigno and Woodford (2003). Further details are provided there and in Woodford (2003, chaps. 3-4). The goal of policy is assumed to be the maximization of the level of expected utility of a representative household. In our model, each household seeks to maximize

$$U_{t_0} \equiv E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\tilde{u}(C_t; \xi_t) - \int_0^1 \tilde{v}(H_t(j); \xi_t) dj \right], \tag{1.1}$$

where C_t is a Dixit-Stiglitz aggregate of consumption of each of a continuum of differentiated goods,

$$C_t \equiv \left[\int_0^1 c_t(i)^{\frac{\theta}{\theta-1}} di \right]^{\frac{\theta-1}{\theta}}, \tag{1.2}$$

with an elasticity of substitution equal to $\theta > 1$, and $H_t(j)$ is the quantity supplied of labor of type j. Each differentiated good is supplied by a single monopolistically competitive producer. There are assumed to be many goods in each of an infinite number of "industries"; the goods in each industry j are produced using a type of labor that is specific to that industry, and also change their prices at the same time. The representative household supplies all types of labor as well as consuming all types of goods. We follow Benigno and Woodford in assuming the isoelastic functional forms,

$$\tilde{u}(C_t; \xi_t) \equiv \frac{C_t^{1-\tilde{\sigma}^{-1}} \bar{C}_t^{\tilde{\sigma}^{-1}}}{1 - \tilde{\sigma}^{-1}}, \tag{1.3}$$

$$\tilde{v}(H_t; \xi_t) \equiv \frac{\lambda}{1+\nu} H_t^{1+\nu} \bar{H}_t^{-\nu}, \tag{1.4}$$

where $\tilde{\sigma}, \nu > 0$, and $\{\bar{C}_t, \bar{H}_t\}$ are bounded exogenous disturbance processes. (We use the notation ξ_t to refer to the complete vector of exogenous disturbances, including $\bar{C}_t$ and $\bar{H}_t$.)

We assume a common technology for the production of all goods, in which (industry-specific) labor is the only variable input,

$$y_t(i) = A_t f(h_t(i)) = A_t h_t(i)^{1/\phi},$$

where A_t is an exogenously varying technology factor, and $\phi > 1$. Inverting the production function to write the demand for each type of labor as a function of the quantities produced of the various differentiated goods, and using the identity

$$Y_t = C_t + G_t$$

to substitute for C_t, where G_t is exogenous government demand for the composite good, we can write the utility of the representative household as a function of the expected production plan $\{y_t(i)\}$.

The producers in each industry fix the prices of their goods in monetary units for a random interval of time, as in the model of staggered pricing introduced by Calvo (1983). We let $0 \leq \alpha < 1$ be the fraction of prices that remain unchanged in any period. Each supplier that changes its price in period t optimally chooses the same price p_t^* that depends on aggregate conditions at the time. Benigno and Woodford (2003) show that the optimal relative price is given by

$$\frac{p_t^*}{P_t} = \left(\frac{K_t}{F_t}\right)^{\frac{1}{1+\omega\theta}}, \tag{1.5}$$

where $\omega \equiv \phi(1+\nu) - 1 > 0$ is the elasticity of real marginal cost in an industry with respect to industry output, and F_t and K_t are two sufficient statistics for aggregate conditions at date t; each is a function of current aggregate output Y_t, the current tax rate τ_t, the current exogenous state ξ_t, and the expected future evolution of inflation, output, taxes and disturbances. In the model of Benigno and Woodford (2003), the tax rate τ_t is a proportional tax on sales revenues, included in the posted price of goods, like a VAT. This tax is the sole source of government revenues; it distorts the allocation of resources owing to its effect on the price-setting decisions of firms.[4]

The Dixit-Stiglitz price index P_t then evolves according to a law of motion

$$P_t = \left[(1-\alpha)p_t^{*1-\theta} + \alpha P_{t-1}^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{1.6}$$

Substitution of (1.5) into (1.6) implies that equilibrium inflation in any period is given by

$$\frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha} = \left(\frac{F_t}{K_t}\right)^{\frac{\theta-1}{1+\omega\theta}}, \tag{1.7}$$

where $\Pi_t \equiv P_t/P_{t-1}$. This defines a short-run aggregate supply relation between inflation and output, given the current tax rate τ_t, current disturbances ξ_t, and expectations regarding future inflation, output, taxes and disturbances.

Again following Benigno and Woodford, we abstract here from any monetary frictions that would account for a demand for central-bank liabilities that earn a substandard rate of return; we nonetheless assume that the central bank can control the riskless short-term nominal interest rate i_t,[5] which is in turn related to other financial asset prices through the arbitrage relation

$$1 + i_t = [E_t Q_{t,t+1}]^{-1},$$

where $Q_{t,T}$ is the stochastic discount factor by which financial markets discount random nominal income in period T to determine the nominal value of a claim to such income in period t. In equilibrium, this discount factor is given by

$$Q_{t,T} = \beta^{T-t}\frac{\tilde{u}_c(C_T;\xi_T)}{\tilde{u}_c(C_t;\xi_t)}\frac{P_t}{P_T}, \tag{1.8}$$

so that the path of nominal interest rates implied by a given path for aggregate output and inflation is given by

$$1 + i_t = \beta^{-1}\frac{\tilde{u}_c(Y_t - G_t;\xi_t)P_t^{-1}}{E_t[\tilde{u}_c(Y_{t+1} - G_{t+1};\xi_{t+1})P_{t+1}^{-1}]}. \tag{1.9}$$

[4] We discuss the consequences for our analysis of allowing for other kinds of taxes in section 2.2. The main limitation on our analysis that results from assuming that only a VAT rate can be varied is that this tax instrument affects aggregate supply incentives but not the intertemporal Euler equation (1.9) for the timing of private expenditure. In fact, many other important instruments of fiscal policy, such as variations in a payroll tax rate or in the rate of tax on labor income, would have the same property, and we think that the supply-side effects of variations in tax policy are the ones of greatest importance in practice. We do however discuss in section 2.2 the conditions under which tax policy can be used to affect the timing of expenditure.

[5] For discussion of how this is possible even in a "cashless" economy of the kind assumed here, see Woodford (2003, chapter 2).

Without entering into the details of how the central bank implements a desired path for the short-term nominal interest rate, it is important to note that it will be impossible for it to ever be driven negative, as long as private parties have the option of holding currency that earns a zero nominal return as a store of value. Hence the zero lower bound

$$i_t \geq 0 \tag{1.10}$$

is a constraint on the set of possible equilibria that can be achieved by any monetary policy. Benigno and Woodford assume that this constraint never binds under the optimal policies that they consider, so that they do not need to introduce any additional constraint on the possible paths of output and prices associated with a need for the chosen evolution of prices to be consistent with a non-negative nominal interest rate. This can be shown to be true in the case of small enough disturbances, given that the nominal interest rate is equal to $\bar{r} = \beta^{-1} - 1 > 0$ under the optimal policy in the absence of disturbances; but it need not be true in the case of larger disturbances. The goal of the present paper is to consider the implications of this constraint.

Our abstraction from monetary frictions, and hence from the existence of seignorage revenues, does not mean that monetary policy has no fiscal consequences, for interest-rate policy and the equilibrium inflation that results from it have implications for the real burden of government debt. For simplicity, we shall assume that all public debt consists of riskless nominal one-period bonds. The nominal value B_t of end-of-period public debt then evolves according to a law of motion

$$B_t = (1 + i_{t-1})B_{t-1} + P_t s_t, \tag{1.11}$$

where the real primary budget surplus is given by[6]

$$s_t \equiv \tau_t Y_t - G_t - \zeta_t. \tag{1.12}$$

Here τ_t, the share of the national product that is collected by the government as tax revenues

[6] Benigno and Woodford (2003) also allow for exogenous variations in the size of government transfer programs, but we do not consider this form of disturbance here.

in period t, is the key fiscal policy decision each period; both the real value of government purchases G_t and the real value of government transfers ζ_t are treated as exogenously given.

Rational-expectations equilibrium requires that the expected path of government surpluses must satisfy an intertemporal solvency condition

$$b_{t-1}\frac{P_{t-1}}{P_t} = E_t \sum_{T=t}^{\infty} R_{t,T} s_T \tag{1.13}$$

in each state of the world that may be realized at date t, where $R_{t,T} \equiv Q_{t,T}P_T/P_t$ is the stochastic discount factor for a real income stream, and This condition restricts the possible paths that may be chosen for the tax rate $\{\tau_t\}$. Monetary policy can affect this constraint, however, both by affecting the period t inflation rate (which affects the left-hand side) and (in the case of sticky prices) by affecting the discount factors $\{R_{t,T}\}$. Again using (1.8), we can equivalently write this as a constraint on the possible paths of aggregate prices, output and the tax rate,

$$b_{t-1}\tilde{u}_c(Y_t - G_t; \xi_t)\Pi_t^{-1} = E_t \sum_{T=t}^{\infty} \beta^{T-t}\tilde{u}_c(Y_T - G_T; \xi_T)[\tau_T Y_T - G_T]. \tag{1.14}$$

The complete set of restrictions on the joint evolution of the variables $\{\Pi_t, Y_t, i_t, \tau_t\}$ under any possible monetary and fiscal policies is then given by equations (1.7), (1.9), (1.10), and (1.13), each of which must hold for each $t \geq t_0$, given the initial public debt b_{t_0-1}. We wish to consider the state-contingent evolution of these variables that will maximize the welfare of the representative household, measured by (1.1), given the exogenously specified evolution of the various disturbances $\{\xi_t\}$.

1.2 A Linear-Quadratic Approximation

Benigno and Woodford (2003) derive a local approximation to the above policy problem that will be of use in our own analysis of optimal policy as well. This is obtained from Taylor series expansions of both the objective and the constraints (other than the zero lower bound, that they assume not to bind) around the steady state values of the endogenous variables that represent an optimal policy in the case that there are no disturbances. They show that this optimal steady state involves zero inflation (and hence identical, constant prices in all industries), an arbitrary constant level of real public debt $\bar{b}$ (that depends on the initial level of real claims on the government), and a constant tax rate $\bar{\tau}$ and output level $\bar{Y}$ that are jointly consistent with the aggregate supply relation (1.7) and the government's budget constraint, given a zero inflation rate and the constant debt level $\bar{b}$. The relation between the steady-state values of these variables implied by the government budget constraint is simply

$$(1-\beta)\bar{b} = \bar{\tau}\bar{Y} - \bar{G} - \bar{\zeta}.$$

Because there is zero inflation in the steady state, the steady-state output level $\bar{Y}$ is just the flexible-price equilibrium level of output (in the absence of disturbances) in the case of a constant tax rate $\bar{\tau}$.

A critical issue for the characterization of optimal stabilization policy is the degree of efficiency of the steady-state level of output $\bar{Y}$ (*i.e.*, the size of the discrepancy between $\bar{Y}$ and the level of output that would maximize utility subject to the feasibility constraint implied by the production technology). The degree of inefficiency of the steady state output level can be measured by the parameter

$$\Phi \equiv 1 - \frac{\theta-1}{\theta}\frac{1-\bar{\tau}}{\bar{\mu}^w} < 1, \tag{1.15}$$

which indicates the steady-state wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labor. (Here $\bar{\mu}^w \geq 1$ is the steady-state level of the markup of wage demands over those associated with competitive labor supply.) Our numerical examples assume that $\bar{b} \geq 0$, implying that $\bar{\tau} \geq 0$ and hence that $\Phi > 0$, so that steady-state output is inefficiently low.[7] This implies that the effects of stabilization policy on the average level of output matter for the welfare evaluation of alternative policies.

Benigno and Woodford (2003) show that it is nonetheless possible to correctly evaluate welfare under alternative policies, to second order in a bound on the amplitude of the exogenous disturbances, using only a log-linear approximation to the model equilibrium relations

[7]This contrasts with the assumption made in Eggertsson and Woodford (2003). However, we nonetheless obtain a quadratic loss function of the form assumed in our previous analysis, as explained below.

to characterize equilibrium dynamics under a given policy. This is possible when one uses as one's welfare measure a quadratic loss function in which the effects of stabilization policy on average output have already been taken into account in the loss function, so that the loss function is purely quadratic, rather than depending explicitly on the average level of output. In fact, Benigno and Woodford show that a quadratic approximation to the expected discounted utility of the representative household is a decreasing function of the objective

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \frac{1}{2} q_\pi \pi_t^2 + \frac{1}{2} q_y (\hat{Y}_t - \hat{Y}_t^*)^2 \right\}, \tag{1.16}$$

where the coefficients q_π, q_y are functions of the model parameters, and the target level of output $\hat{Y}_t^*$ is a function of all of the exogenous real disturbances (discussed further in the next section). In the case that both the share of output consumed by the government and the steady-state tax rate are not extremely large, the coefficients q_π, q_y are shown both to be positive; this is true for the numerical calibrations considered below. Hence the stabilization of both inflation and the welfare-relevant output gap $y_t \equiv \hat{Y}_t - \hat{Y}_t^*$ is desirable for welfare.

Given the purely quadratic form of the objective (1.16), a log-linear approximation to the model structural relations suffices to allow a characterization of welfare under alternative rules that is accurate to second order, and hence a characterization of optimal policy that is accurate to first order in the amplitude of the disturbances. A first-order Taylor series expansion of (1.7) around the zero-inflation steady state yields the log-linear aggregate-supply relation

$$\pi_t = \kappa[\hat{Y}_t + \psi \hat{\tau}_t + c'_\xi \xi_t] + \beta E_t \pi_{t+1}, \tag{1.17}$$

where π_t is the inflation rate, $\hat{Y}_t \equiv \log(Y_t/\bar{Y})$, and $\hat{\tau}_t \equiv \tau_t - \bar{\tau}$.[8] Here the coefficients are given by

$$\kappa \equiv \frac{(1-\alpha\beta)(1-\alpha)}{\alpha} \frac{\omega + \sigma^{-1}}{1 + \omega\theta} > 0,$$

[8]Note a difference in notation from that used in Benigno and Woodford (2003), where $\hat{\tau}_t$ refers to the deviation of $\log \tau_t$ from its steady-state value. Here we wish to be able to consider the case of a zero steady-state tax rate. Our alternative notation implies a corresponding difference in the value of the coefficient ψ; the coefficient ψ defined in Benigno and Woodford (2003) is equal to $\bar{\tau}\psi$ in our notation. Note that our coefficient ψ has a positive value even in the case that $\bar{\tau} = 0$, while the coefficient defined by Benigno and Woodford is zero in that case, even though an increase in the tax rate will still shift the aggregate-supply relation in that case.

$$\psi \equiv \frac{1}{1-\bar{\tau}} \frac{1}{\omega + \sigma^{-1}} > 0,$$

where $\sigma \equiv \tilde{\sigma}\bar{C}/\bar{Y} > 0$ is an intertemporal elasticity of substitution for total (as opposed to merely private) expenditure.[9]

This is the familiar "New Keynesian Phillips curve" relation, extended here to take account of the effects of variations in the level of distorting taxes on supply costs. (Note that $-c'_\xi \xi_t - \psi \hat{\tau}_t$ represents the log deviation of the flexible-price equilibrium level of output from the steady-state output level $\bar{Y}$, in the case of real disturbances ξ_t and a tax rate $\hat{\tau}_t$.) It is useful to write this approximate aggregate-supply relation in terms of the welfare-relevant output gap y_t. Equation (1.17) can be equivalently be written as

$$\pi_t = \kappa[y_t + \psi\hat{\tau}_t + u_t] + \beta E_t \pi_{t+1}, \tag{1.18}$$

where u_t is composite "cost-push" disturbance, indicating the degree to which the various exogenous disturbances included in ξ_t preclude simultaneous stabilization of inflation, the welfare-relevant output gap, and the tax rate. (The effects of real disturbances on this term are discussed in the next section.) Alternatively we can write

$$\pi_t = \kappa[y_t + \psi(\hat{\tau}_t - \hat{\tau}_t^*)] + \beta E_t \pi_{t+1}, \tag{1.19}$$

where $\hat{\tau}_t^* \equiv -\psi^{-1} u_t$ indicates the tax change needed at any time to offset the"cost-push" shock, in order to allow simultaneous stabilization of inflation and the output gap (the two stabilization objectives reflected in (1.16)).

The other constraint on possible equilibrium paths considered by Benigno and Woodford (2003) is the intertemporal government solvency condition. A log-linear approximation to (1.14) can be written in the form

$$\hat{b}_{t-1} - s_b \pi_t - s_b \sigma^{-1} y_t = -f_t \ + E_t \sum_{T=t}^{\infty} \beta^{T-t} [b_y y_T + b_\tau (\hat{\tau}_T - \hat{\tau}_T^*)], \tag{1.20}$$

where $\hat{b}_t \equiv (b_t - \bar{b})/\bar{Y}$ measures the deviation of the real public debt from its steady-state level (as a fraction of steady-state output),[10] and f_t is a composite measure of exogenous

[9]Under the simplifying assumption of zero government purchases, maintained in our numerical examples below, σ is simply the preference parameter $\tilde{\sigma}$.

"fiscal stress." (Note that the sum $\hat{b}_{t-1} + f_t$ indicates the degree to which a plan to maintain zero inflation and a zero output gap for all periods $T \geq t$ would fail to be consistent with government solvency. The way in which real disturbances affect the term f_t is discussed further in the next section.) The coefficient $s_b \equiv \bar{b}/\bar{Y}$ indicates the steady-state level of public debt as a proportion of steady-state output. Under the simplifying assumption of zero government purchases, maintained in our numerical examples, the coefficients b_y, b_τ, indicating the effect on the government budget of variations in aggregate output and the tax rate respectively, are equal to[11]

$$b_y = (1 - \sigma^{-1})\bar{\tau}, \qquad b_\tau = 1.$$

In deriving the first-order conditions that characterize optimal policy, it is useful to write this constraint in a flow form. Note that if (1.20) holds each period, it follows that

$$\hat{b}_{t-1} - s_b\pi_t - s_b\sigma^{-1}y_t + f_t = [b_y y_t + b_\tau(\hat{\tau}_t - \hat{\tau}_t^*)] + \beta E_t[\hat{b}_t - s_b\pi_{t+1} - s_b\sigma^{-1}y_{t+1} + f_{t+1}] \quad (1.21)$$

each period as well. The solvency condition also implies the transversality condition

$$\lim_{T\to\infty} \beta^T E_t[\hat{b}_{T-1} - s_b\pi_T - s_b\sigma^{-1}y_T + f_T] = 0,$$

and this transversality condition, together with the requirement (1.21) for each period, implies (1.20).

The linear-quadratic policy problem considered by Benigno and Woodford (2003) is then the choice of state-contingent paths for the endogenous variables $\{\pi_t, y_t, \hat{\tau}_t, \hat{b}_t\}$ from some date t_0 onward so as to minimize the quadratic loss function (1.16), subject to the constraint that conditions (1.19) and (1.20) be satisfied each period, given an initial value $\hat{b}_{t_0-1}$ and subject also to the constraints that π_{t_0} and y_{t_0} equal certain precommitted values,

$$\pi_{t_0} = \bar{\pi}_{t_0}, \qquad y_{t_0} = \bar{y}_{t_0}, \quad (1.22)$$

[10]Here again our notation differs from that of Benigno and Woodford (2003), so that we can treat the case of a steady state with zero public debt. As a consequence, our coefficients b_y, b_τ are equal to $(1-\beta)s_b$ times the definitions given by Benigno and Woodford.

[11]More general expressions for these coefficients can be found in the appendix to Benigno and Woodford (2003), taking account of the change in notation discussed in the previous footnote.

that may depend on the state of the world in period t_0. The allowance for appropriately chosen initial constraints allows us to ensure that the policy judged to be optimal from some date t_0 onward corresponds to the commitment that would have optimally been chosen at some earlier date. This means that even if we suppose that at date t_0 it is already known that a disturbance has occurred, the optimal policy response that is computed is the way that a policymaker should have committed in advance to respond to such a shock, rather than one that takes advantage of the opportunity to choose an optimal policy afresh and exploit existing expectations.[12]

1.3 The Natural Rate of Interest and the Zero Bound

In the Benigno - Woodford (2003) characterization of optimal monetary and fiscal policy, it is not necessary to include among the constraints of the policy problem any relations that connect interest rates to the target variables (inflation and the output gap). It suffices that there be *some* feasible level of short-term nominal interest rate at each point in time associated with the solution to the constrained optimization problem that they define; it does not actually matter what this interest rate is, in order to determine the optimal state-contingent paths of inflation, output, tax rates, and the public debt. And since the nominal interest rate is positive in the optimal steady state, the solution to their optimization problem continues to imply a positive nominal interest rate at all times, as long as shocks are small enough.

Here, however, we are interested in the case in which there are occasionally disturbances large enough to cause the zero bound to bind, though we shall continue to assume that the above local approximations to both the model structural relations and the welfare objective are sufficiently accurate. In order to see how possible paths for the target variables are restricted by this constraint, it is necessary to consider the equilibrium relation between

[12] Thus we consider a policy that is "optimal from a timeless perspective," in the sense defined in Woodford (2003, chap. 7). In fact, in the numerical exercises reported below, the optimal responses that are reported are the same as those that would be obtained if the initial constraints (1.22) were omitted, since we set the initial lagged Lagrange multipliers equal to zero.

interest rates and aggregate expenditure. A log-linear approximation to the Euler equation (ISexact) for optimal expenditure can be written in the form[13]

$$\hat{Y}_t - g_t = E_t[\hat{Y}_{t+1} - g_{t+1}] - \sigma(i_t - E_t\pi_{t+1} - \bar{r}), \tag{1.23}$$

where g_t is a composite exogenous disturbance indicating the percentage change in period t output required in order to hold constant the representative household's marginal utility of additional real expenditure (despite shifts in impatience to consume or in government purchases), σ is again the intertemporal elasticity of substitution of private expenditure, and $\bar{r} \equiv \beta^{-1} - 1 > 0$ is the steady-state real rate of interest.

This can alternatively be written in terms of the welfare-relevant output gap as

$$y_t = E_t y_{t+1} - \sigma(i_t - E_t\pi_{t+1} - r_t^n), \tag{1.24}$$

where

$$r_t^n \equiv \bar{r} + \sigma^{-1}[(g_t - \hat{Y}_t^*) - E_t(g_{t+1} - \hat{Y}_{t+1}^*)] \tag{1.25}$$

represents the "natural rate of interest," *i.e.*, the equilibrium real rate of interest at each point in time that would be required in order for output to be kept always at its target level.[14] Note that the natural rate of interest depends only on exogenous real disturbances.[15] It indicates the degree to which short-term nominal interest rates must be adjusted in order to be consistent with full achievement of both stabilization goals — *i.e.*, in order for output to equal the target level while inflation is equal to zero each period. If the natural rate

[13]The i_t in this equation actually refers to $\log(1 + i_t)$ in the notation of section 1.1, *i.e.*, to the log of the gross nominal interest yield on a one-period riskless investment, rather than to the net one-period yield. Note also that this variable, unlike the others appearing in our log-linear approximate relations, is not defined as a deviation from a steady-state value. This is so that we can continue to write the zero bound as a simple requirement that i_t be non-negative. Hence the steady-state value $\bar{r}$ appears in equation (1.23).

[14]For symmetry with our definition of i_t, we have also defined r_t to be the absolute level of the natural rate — technically, the log of a gross real rate of return — rather than a deviation from the steady-state natural rate $\bar{r}$.

[15]The natural rate of interest defined here does not correspond to the flexible-price equilibrium real rate of interest, which would depend on the path of the tax rate rather than only on the exogenous disturbances. However, in the case of isoelastic preferences (assumed here) and zero government purchases (assumed in our numerical example), it does correspond to what the equilibrium real rate of interest would be under flexible prices in the event that the tax rate τ_t were maintained at the steady-state level.

of interest is sometimes negative, the zero lower bound on nominal interest rates alone will imply that full achievement of these stabilization objectives is impossible, even in the absence of cost-push shocks.

Taking account of the zero bound thus requires that we adjoin to the set of constraints considered by Benigno and Woodford (2003) two additional constraints, namely (1.24) and the zero bound

$$i_t \geq 0. \tag{1.26}$$

We can replace these by a single constraint on possible paths for inflation and the output gap,

$$y_t \leq E_t y_{t+1} + \sigma(r_t^n + E_t \pi_{t+1}), \tag{1.27}$$

as in Eggertsson and Woodford (2003). The optimal policy problem is then to choose state-contingent paths $\{\pi_t, y_t, \hat{\tau}_t, \hat{b}_t\}$ to minimize (1.16) subject to the constraints that (1.19), (1.20) and (1.27) be satisfied each period, together with the initial constraints (1.22). This reduces to the problem considered by Benigno and Woodford (2003) in the event that (1.27) never binds. It is clear that the constraint is a tighter one the lower the value of the natural rate of interest.

There are various reasons why real disturbances may shift the natural rate of interest. On the one hand, there may be temporary fluctuations in the factor g_t appearing in (1.23). These may result either from variations in government purchases G_t, or from variations in the preference parameter $\bar{C}_t$, indicating the level of private expenditure required to maintain a constant marginal utility of real expenditure, and hence the variations in private expenditure that occur if the private sector smooths the marginal utility of expenditure. But on the other hand, any source of temporary fluctuations in the target output level $\hat{Y}_t^*$ will also imply variation in the natural rate of interest as defined here. As Benigno and Woodford (2003) show, a large variety of real disturbances should affect $\hat{Y}_t^*$, including (in the case of a distorted steady state) variations in market power, as well as disturbances to both preferences and technology.

In the baseline case considered in the next section, we shall consider the challenges for policy created by fluctuations in the natural rate of interest, while abstracting from the effects of variations in either the cost-push term u_t in (1.18) or in the fiscal stress term f_t in (1.20). It is possible for a real disturbance to affect r_t^n without any effect on either u_t or f_t. In our numerical examples, we shall simplify by assuming that there are no government purchases, and we consider the effects of exogenous variations in the factors $\bar{C}_t$ and $\bar{H}_t$ in (1.3) – (1.4), or in the technology factor A_t. Variation in the factor $\bar{C}_t$ results in variation in the term g_t;[16] indeed, under the simplifying assumption of no government purchases, g_t is just the deviation of $\log \bar{C}_t$ from its steady-state value. At the same time, all three disturbances effect the target level of output, which under the assumption of no government purchases is given by[17]

$$\hat{Y}_t^* = \frac{\sigma^{-1}}{\omega + \sigma^{-1}} g_t + \frac{\omega}{\omega + \sigma^{-1}} q_t. \tag{1.28}$$

Here q_t is the increase in log output that would be required to maintain a constant marginal disutility of labor effort; it is positive if $\bar{H}_t$ or A_t are temporarily above their steady-state levels.

In the case assumed in our baseline analysis, an exogenous disturbance temporarily makes both g_t and q_t negative, but reduces g_t by more. It follows from (1.28) that $\hat{Y}*_t$ declines, but by less than the decline in g_t, so that $g_t - \hat{Y}_t^*$ is temporarily negative. Hence a temporary disturbance of the kind proposed implies a temporary decline in the natural rate of interest. In the specific numerical exercises reported below, we assume that an exogenous disturbance changes $\bar{C}_t$, $\bar{H}_t$ and/or A_t from their steady-state levels, after which there is a probability $0 < \rho < 1$ each period that $\bar{C}_{t+1} = \bar{C}_t$, $\bar{H}_{t+1} = \bar{H}_t$, and $A_{t+1} = A_t$, and on the other hand a probability $1 - \rho$ that $\bar{C}_{t+1} = \bar{C}$, $\bar{H}_t = \bar{H}$, and $A_t = \bar{A}$. Once the exogenous

[16] In our model, the factor $\bar{C}_t$ is treated as a parameter of the preferences of the representative household. However, the variable C_t stands for all private expenditure in our model, and the utility function $u(C_t; \xi_t)$ is actually to be understood as a reduced-form representation of the way in which utility is increased by real private expenditure of all types — on investment as well as consumer goods. (See Woodford, 2003, chap. 4, for further discussion.) Thus fluctuations in $\bar{C}_t$ might also represent fluctuations in the marginal efficiency of investment spending, for reasons treated as exogenous to our model.

[17] This is a special case of the more general formula given in the appendix of Benigno and Woodford (2003).

preference and technology factors return to their steady-state levels, they are expected to remain permanently at those values. In the case of this kind of disturbance, $E_t g_{t+1} = \rho g_t$ and $E_t q_{t+1} = \rho q_t$, so that (1.25) implies that

$$r_t^n = \bar{r} + (1-\rho)\frac{\omega\sigma^{-1}}{\omega + \sigma^{-1}}(g_t - q_t).$$

Thus r_t^n falls below its steady-state level $\bar{r} > 0$ when a shock of the kind hypothesized occurs (so that $g_t < q_t < 0$), remains at the lower level for as long as the exogenous factors remain at their irregular values, and returns to the steady-state level $\bar{r}$ (permanently) once these factors return to their normal values. If the temporary disturbance is large enough (or temporary enough), the natural rate of interest may be negative during the period that these factors depart from their steady-state values. This is the case considered in the numerical exercises below.

Benigno and Woodford (2003) show that if there are no government purchases, disturbance of these kinds have no cost-push effect, *i.e.*, that $u_t = 0$. This occurs because the variations in $\bar{C}_t$, $\bar{H}_t$ and A_t shift the flexible-price, constant-tax-rate equilibrium level of output to exactly the same extent as the target level of output $\hat{Y}_t^*$. As a consequence, $\hat{Y}_t = \hat{Y}_t^*$ at all times is consistent with a constant tax rate τ_t under flexible prices, and consequently also consistent with a constant tax rate in the event that inflation is zero at all times, even in the presence of price stickiness. Thus the aggregate-supply relation (1.19) requires no variation in tax rates in order for complete achievement of both stabilization goals despite the occurrence of shocks of this kind.

However, complete stabilization of inflation and the output gap may nonetheless be inconsistent with intertemporal government solvency. On the one hand, because the disturbance reduces the target level of output $\hat{Y}_t^*$, as discussed above, it reduces the level of real government revenues associated with an unchanged tax rate. On the other hand, a reduction in the real rate of interest associated with zero inflation and a zero output gap (*i.e.*, the natural rate of interest), for the reasons also just discussed, will reduce the size of the tax revenues needed for government solvency. It is possible (though a rather special case) that these countervailing effects may precisely cancel. In the case of a disturbance of the specific (Markovian) kind discussed above, the effect on the left-hand side of (1.14) is a percentage increase equal to

$$\sigma^{-1}(g_t - \hat{Y}_t^*),$$

while the effect on the right-hand side is a percentage increase equal to

$$\frac{1-\beta}{1-\beta\rho}[\hat{Y}_t^* + \sigma^{-1}(g_t - \hat{Y}_t^*)].$$

Intertemporal solvency continues to be satisfied without any change in the tax rate if these two expressions are equal.

Using (1.28), we see that this occurs if it happens that

$$\left[\omega - \frac{\beta^{-1}-1}{1-\rho}\right]\sigma^{-1}g_t = \left[\sigma^{-1} + \frac{\beta^{-1}-1}{1-\rho}\right]\omega q_t. \tag{1.29}$$

In the baseline example in the next section, we shall assume a disturbance in which the relative magnitudes of the shifts in g_t and in q_t are precisely those needed for this to be so, so that $f_t = 0$. (Alternatively, we assume a degree of persistence of the disturbance of precisely the size needed to satisfy this condition, given the relative magnitudes of the shifts in the two types of exogenous factors.) We assume parameter values under which both factors in square brackets are positive; hence (1.29) requires that g_t and q_t have the same sign, as assumed above. One can show that it also implies that g_t is larger than q_t in absolute value, so that r_t^n moves in the same direction as g_t and q_t, as also asserted above. In fact, one can show that (1.29) implies that

$$r_t^n = \bar{r} + (\beta^{-1} - 1)\hat{Y}_t^*. \tag{1.30}$$

It follows that in our baseline example, the real disturbance that temporarily changes the natural rate of interest has no effect on either the cost-push term u_t in (1.18) or the fiscal stress term f_t in (1.20). This means that as long as the natural rate of interest remains always non-negative, optimal monetary and fiscal policy would involve a constant tax rate, a zero inflation rate and a zero output gap, and a nominal interest rate that tracks the temporary variation in the natural rate of interest. However, if the natural rate of interest is temporarily negative, it will not be possible to achieve such an equilibrium. In this case, the zero bound is a binding constraint. We take up the characterization of optimal policy in such a case in the next section.

In general, of course, there is no reason for (1.29) to happen to hold. The case of most practical interest is one in which a real disturbance that temporarily lowers the natural rate of interest also lowers the fiscal stress term f_t in (1.20). The reason is that the natural rate of interest is only negative in the event of a disturbance that has a particularly large effect on the natural rate of interest. This is most likely in the case that the shift in $\hat{Y}_t^*$ does not offset the shift in g_t to the extent required for (1.29) to hold; but this means that the most likely case is one in which the effect on the government budget of the decline (if any) in $\hat{Y}*_t$ is not as great as the effect of the decline in the real interest rate associated with complete inflation and output-gap stabilization. Hence it is most likely in practice that a sharp decline in the natural rate of interest will be associated with a reduction in fiscal stress (a negative value of f_t).

In section 2.3, we consider an alternative form of disturbance with this feature. Specifically, we consider the case of a disturbance which lowers the natural rate of interest without affecting the target level of output. (An example of such a disturbance would be a temporary decline in the rate of time preference; this is equivalent to a simultaneous reduction in $\bar{C}_t$ and increase in $\bar{H}_t$. Because the intratemporal first-order condition for optimal labor supply is unaffected by such a disturbance, the flexible-price equilibrium level of output is unaffected. And in the case of zero government purchases, this implies that $\hat{Y}_t^*$ is unaffected as well.) In this alternative special case, the fiscal stress term is given by

$$f_t = s_b \sum_{T=t}^{\infty} \beta^{T-t+1} E_t[r_T^n - \bar{r}]. \tag{1.31}$$

Hence a disturbance that temporarily lowers the natural rate of interest results in a reduction in fiscal stress. In section 2.3, we consider how the character of optimal policy changes in this more complex case.

1.4 The Zero Lower Bound as a Constraint on Stabilization Policy: A Simple Example

Here we show that the existence of the zero lower bound on nominal interest rates can have important consequences for macroeconomic stability. We do this by considering policy rules that would lead to optimal outcomes in the event that real disturbances are not large enough to cause the zero lower bound to bind, and then showing that in the event of a larger disturbance, the bound can bind. We further show that when the bound does bind, the consequence can be both substantial deflation and a substantial negative output gap, both of which are have adverse welfare consequences under the analysis of stabilization objectives presented above.

Our analysis is simplest if we assume an initial public debt of zero, as well as zero government purchases and zero government transfers at all times, so that the steady-state tax rate $\bar{\tau}$ is also zero. In this zero-debt case, $s_b = 0$, so that the constraint (1.20) takes a simpler form,

$$\hat{b}_{t-1} = E_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{\tau}_T. \tag{1.32}$$

There are no effects of inflation variation on the government budget in this case (to first order), since there is no nominal public debt in the steady state; and there are no effects of output variation (to first order), either, since there is a zero tax rate in the steady state.

There is also no fiscal stress term in (1.32). The assumption that $s_b = 0$ implies that there is no fiscal stress effect of variations in either the natural rate of interest (since there is no debt to roll over, in the steady state) or in the target level of output (output variations have no revenue effects, to first order, because the steady-state tax rate is zero). Hence as long as the real disturbance that lowers the natural rate of interest has no cost-push effect — which will be true of any disturbances to $\bar{C}_t$, $\bar{H}_t$, or A_t, under our assumption that $s_G = 0$ — it will have no fiscal stress effect, either. Hence we can abstract from variations in either u_t or f_t in this case, without requiring the special assumption (1.29).

In the absence of either cost-push or fiscal stress effects, it is evident that (1.16) would be minimized by a policy that maintains $\pi_t = y_t = \hat{\tau}_t = 0$ at all times, as long as this is consistent with the zero bound on interest rates. Such a policy is consistent with (1.26) as long as $r_t^n \geq 0$ at all times, which will be true in the case of small enough real disturbances. On the other hand, (1.26) implies that no such equilibrium is possible in the event that the natural rate of interest is occasionally negative.

This suggests the following thought experiment. Suppose that fiscal and monetary policy are conducted in accordance with the following simple rules: (i) The tax rate $\hat{\tau}_t$ that is chosen at each point in time is the one that the fiscal authority expects to be able to maintain indefinitely, without violating the intertemporal government solvency condition; that is, an expected path of taxes such that $E_t\hat{\tau}_T = \hat{\tau}_t$ for all $T \geq t$ is consistent with (1.20); and (ii) monetary policy is used to ensure that inflation equals zero at all times, unless the zero bound prevents interest rates from being lowered enough to prevent deflation. In our baseline case, since (1.20) reduces to (1.32), the proposed fiscal rule is simply

$$\hat{\tau}_t = (1-\beta)\hat{b}_{t-1}. \tag{1.33}$$

The monetary policy rule (a strict zero inflation target of the kind discussed in Eggertsson and Woodford, 2003) implies that

$$\pi_t \leq 0 \tag{1.34}$$

each period, and that in each period either (1.26) or (1.34) must hold with equality.

If real disturbances are small enough so that the natural rate of interest is non-negative at all times, then the policy rules (1.33) – (1.34) result in an equilibrium in which $\hat{b}_t = \hat{b}_{t_0-1} = 0$ for all $t \geq t_0$, so that $\hat{\tau}_t = 0$ at all times, and in which $\pi_t = 0$ each period, so that $y_t = 0$ at all times as well. (This requires that the nominal interest rate satisfy $i_t = r_t^n$ each period.) This is obviously an optimal equilibrium; so the proposed rules would be optimal (in our baseline case) in the event of any small enough real disturbances.

Let us consider instead the consequences of these policies in the case of a larger disturbance, that temporarily causes the natural rate of interest to be negative. We shall illustrate what can happen using a numerical example, upon which we elaborate in later sections.

	Zero debt	High debt
β	0.99	0.99
σ	0.5	0.5
ω	0.47	0.47
θ	10	10
κ	0.02	0.02
s_G	0	0
s_ζ	0	0.176
s_b	0	2.4
$\bar{\tau}$	0	0.2
ψ	0.40	0.51
$\bar{\mu}^w$	1.08	1.08
Φ	0.17	0.33
q_y/q_π	0.032	0.032
$\bar{r}$	0.04	0.04
$\underline{r}$	-0.02	-0.02
ρ	0.9	0.9

Table 1: Parameter values for the numerical examples.

The numerical parameter values that we assume are given in the first column of Table 1.[18] We assign values to the parameters $\beta, \sigma, \omega, \theta$, and κ in the same way as in Eggertsson and Woodford (2003). In our baseline case, the new parameters $s_b \equiv \bar{b}/\bar{Y}$, $s_G \equiv \bar{G}/\bar{Y}$, and $s_\zeta \equiv \bar{\zeta}/\bar{Y}$ are assigned the values already discussed above. The value of the steady-state wage markup $\bar{\mu}^w$ is taken from Benigno and Woodford (2003), as are the values of s_b and $\bar{\tau}$ in the "high-debt" calibration (discussed in section 2.3 below). The remaining values listed in the table are then the values for other parameters implied by these ones.[19]

[18]Note that periods of our model are interpreted as referring to quarters.

[19]Note that the value quoted for the relative weight q_y/q_π assumes a normalization of the target variables in which the inflation rate is measured in percentage points per annum, while the output gap is measured in percentage points. Note also that while the larger steady-state distortions in the high-debt case affect the absolute magnitudes of q_π and q_y, they do not affect the ratio of the two, which is all that matters for the definition of the optimal policies, assuming that both weights remain positive, as is true in both cases shown in the table.

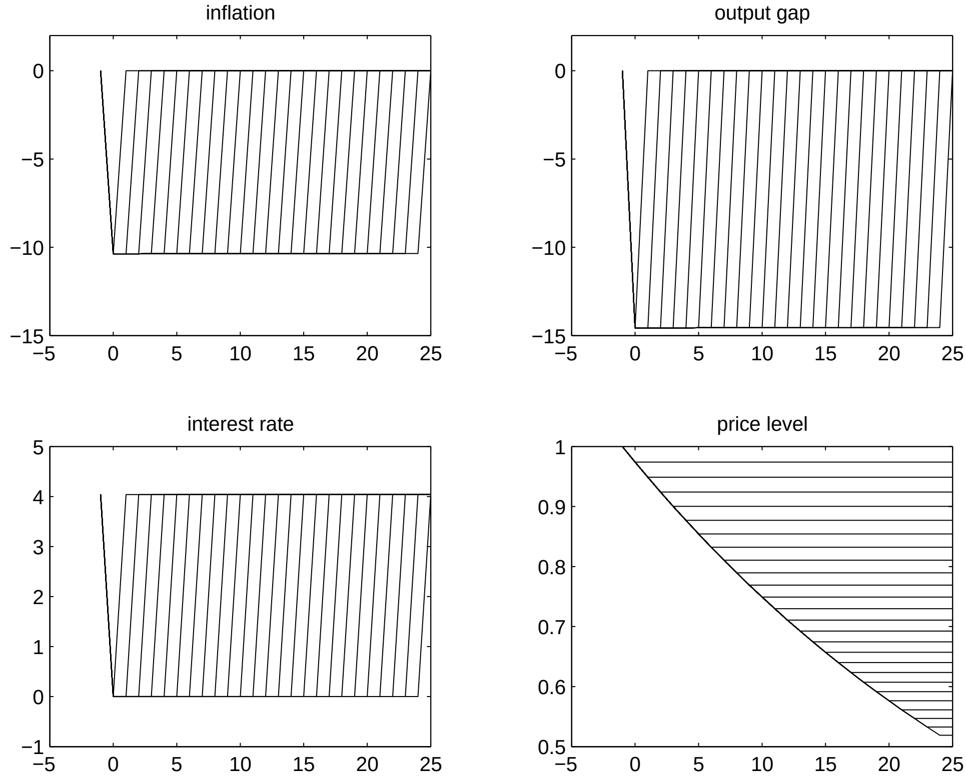

Figure 1: Consequences of a temporary decline in the natural rate of interest: the case of a strict inflation target and tax smoothing.

In our numerical examples, we consider a real disturbance of the following sort. Our parameter values imply that the steady-state natural rate of interest $\bar{r}$ is equal to 4 percent per annum (.01 per quarter). We assume that prior to quarter 0, the economy has been in the optimal steady state. In quarter 0, an unexpected shock lowers the natural rate of interest to a temporary value of $\underline{r}$ = -2 percent per annum. The natural rate then remains at this lower level each quarter with probability $\rho = 0.9$, while with a ten percent probability it returns to the steady-state level. After it returns to the steady-state level, it is expected to remain there thereafter.

In the case of the policy rules (1.33) – (1.34), the occurrence of the disturbance will cause the zero lower bound to bind, and the central bank will be unable to prevent deflation and a negative output gap. However, to first order this has no consequences for the government's budget in our baseline case, and the fiscal rule still implies that $\hat{\tau}_t = 0$ for all t. The consequences for inflation and output are then the same as in Eggertsson and Woodford (2003), where a strict zero inflation target is considered under the assumption of lump-sum taxation (so that the aggregate-supply relation is not shifted by any changes in tax policy). Both π_t and y_t fall to negative levels, which they maintain for as long as $r_t^n = \underline{r}$. Once the natural rate of interest returns to its normal level, at the random date T_1, the zero bound ceases to bind, and monetary policy again succeeds at maintaining zero inflation from that date onward. The price level is again stabilized, but at a permanently lower level than existed prior to the disturbance (and that is lower the longer the disturbance lasts); the output gap is again zero from date T_1 onward. Figure 1 plots the state-contingent paths of inflation, the output gap, the nominal interest rate, and the price level in this solution, for each of the possible realizations of T_1, in the case of the parameter values listed in Table 1. (The figure superimposes the paths for $T_1 = 1, T_1 = 2$, and so on. The same format is used to display the economy's state-contingent evolution under alternative policies below.)

We see from the figure that even a slightly negative natural rate of interest can have dire consequences under a policy rule of this sort,[20] even though the policy is one that might seem reasonable, and that indeed would be optimal in the case of disturbances small enough that the zero bound would not bind. However, these unfortunate consequences of the zero bound can be substantially mitigated, at least in principle, through commitment to monetary and fiscal policies of a more sophisticated type. The kind of policy commitments that are required are treated next.

2 Optimal Policy Commitments

We now turn to the characterization of optimal monetary and fiscal policy commitments, in the case that (1.26) is an additional binding constraint on what policy can achieve. We begin by considering the case in which both monetary and fiscal policy are chosen optimally, and then (in section 3) compare this ideal case to suboptimal alternatives.

[20]During the entire period (of random length) for which the natural rate of interest is negative, deflation occurs at a rate of -10.4 percent per annum, and aggregate output is 14.6 percentage points below its target level.

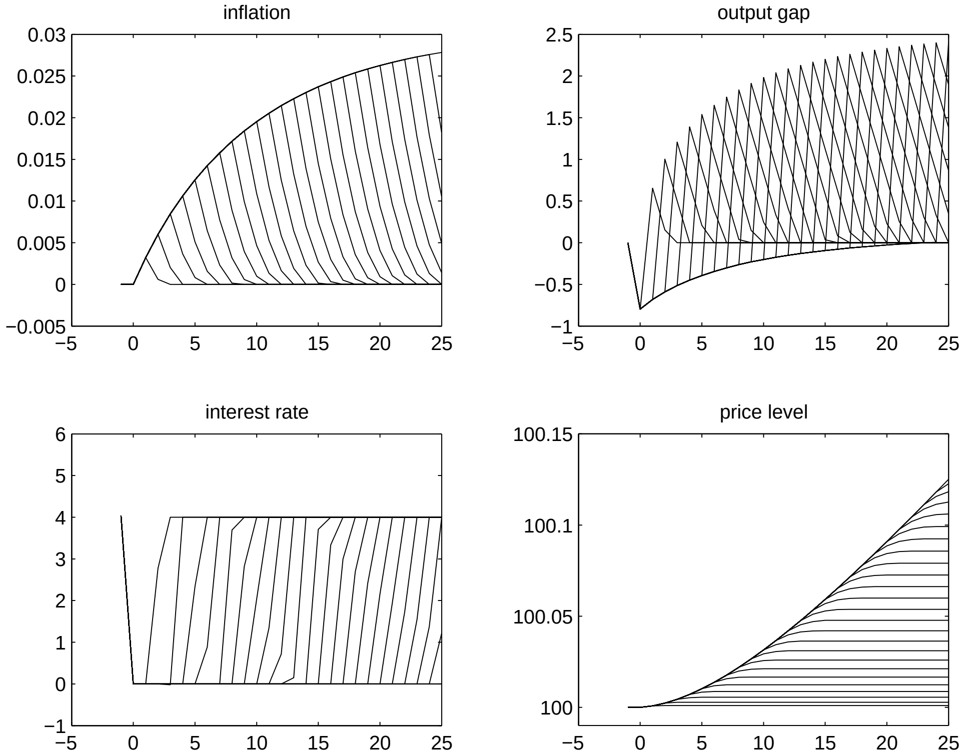

Figure 2: Optimal responses to a decline in the natural rate of interest: the baseline case.

2.1 Optimal Policy with Zero Initial Public Debt

We first consider the baseline model already introduced in section 1.4, characterized by zero initial public debt, zero government purchases, and a zero steady-state tax rate, so that the intertemporal government solvency condition (1.20) reduces to (1.32). We thus consider the problem of choosing state-contingent paths $\{\pi_t, y_t, \hat{\tau}_t, \hat{b}_t\}$ to minimize (1.16) subject to the constraints that (1.19), (1.27), and (1.32) be satisfied each period, together with the initial constraints (1.22), given an initial public debt $\hat{b}_{t_0-1}$. We consider the optimal response to fluctuations in the natural rate of interest r_t^n that affect the tightness of the constraint (1.27), but the term $\hat{\tau}_t^*$ in the constraint (1.19) is assumed always to equal zero.

The Lagrangian for this optimization problem is of the form

$$\begin{aligned}\mathcal{L}_{t_0} \;=\; & E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \{ \frac{1}{2} q_\pi \pi_t^2 + \frac{1}{2} q_y y_t^2 + \varphi_{1t}[y_t - y_{t+1} - \sigma \pi_{t+1}] + \varphi_{2t}[\pi_t - \kappa(y_t + \psi \hat{\tau}_t) - \beta \pi_{t+1}] \\ & + \varphi_{3t}[\hat{b}_{t-1} - \hat{\tau}_t - \beta \hat{b}_t] \} - [\beta^{-1} \sigma \varphi_{1,t_0-1} + \varphi_{2,t_0-1}] \pi_{t_0} - \beta^{-1} \varphi_{1,t_0-1} y_{t_0},\end{aligned}$$

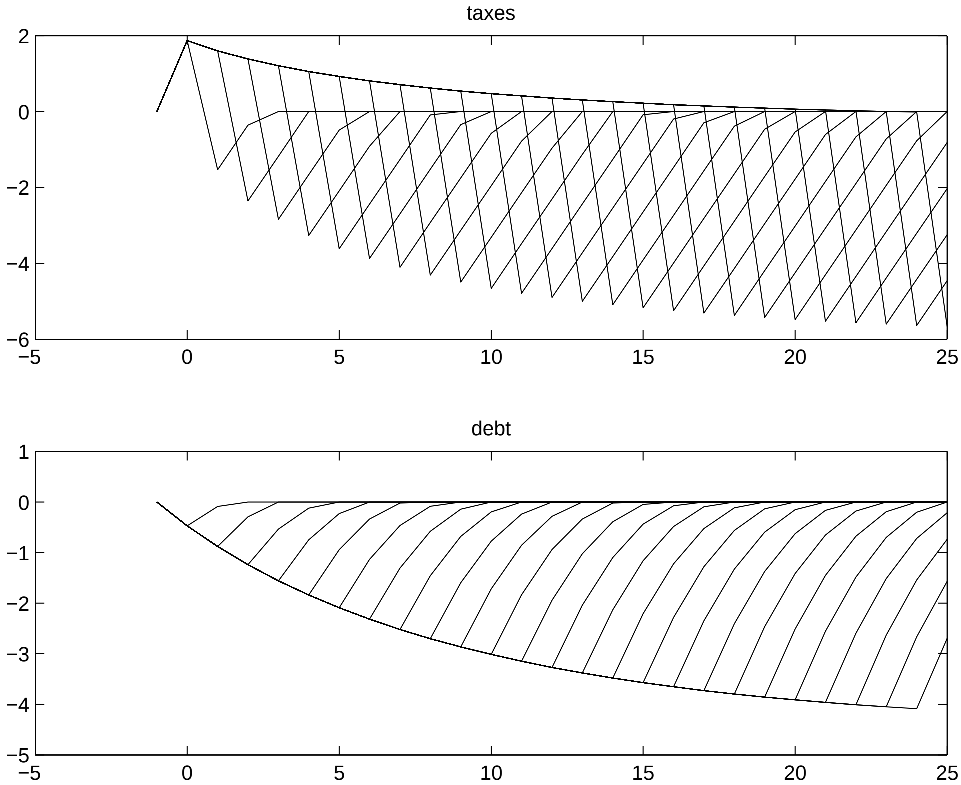

Figure 3: Optimal responses of fiscal variables: the baseline case.

where φ_{1t} is the Lagrange multiplier associated with constraint (1.27), φ_{2t} is the Lagrange multiplier associated with constraint (1.19), and φ_{3t} is the Lagrange multiplier associated with constraint (1.32).[21] The final two terms of the Lagrangian correspond to the initial constraints (1.22).[22] The notation chosen for the multipliers corresponding to these constraints gives the first-order conditions below a time-invariant form; and this interpretation of the multipliers indicates how the values of these multipliers should be determined if optimal policy from t_0 onward is to represent the continuation of an optimal policy chosen at an earlier date.

Differentiation of the Lagrangian leads to the following first-order conditions for an op-

[21]In forming the Lagrangian, we have used the flow form of (1.32), *i.e.*, the relation analogous to (1.21) in the case of a zero-debt economy.

[22]In fact, in the results reported below, we assume the values $\varphi_{1,t_0-1} = \varphi_{2,t_0-1} = 0$, so our conclusions regarding optimal policy are the same as if we were to assume no initial constraints at all.

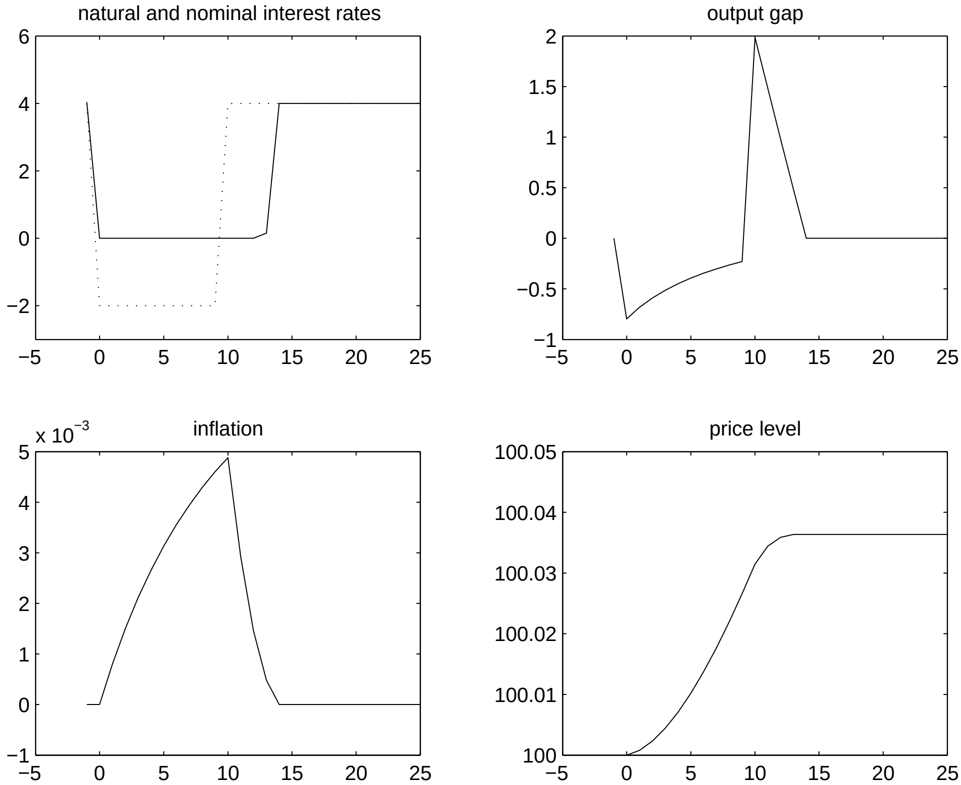

Figure 4: Optimal responses when the disturbance lasts exactly 10 quarters.

timal policy commitment. The equalities

$$q_\pi \pi_t - \beta^{-1}\sigma\varphi_{1,t-1} + \varphi_{2t} - \varphi_{2,t-1} = 0, \tag{2.1}$$

$$q_y y_t + \varphi_{1t} - \beta^{-1}\varphi_{1,t-1} - \kappa\varphi_{2t} = 0, \tag{2.2}$$

$$\kappa\psi\varphi_{2t} + \varphi_{3t} = 0, \tag{2.3}$$

and

$$\varphi_{3t} = E_t\varphi_{3,t+1} \tag{2.4}$$

must hold for each $t \geq t_0$. In addition, the inequality

$$\varphi_{1t} \geq 0 \tag{2.5}$$

must hold for each $t \geq 0$, together with the complementary slackness condition that each period either (1.27) or (2.5) must hold with equality, *i.e.*, φ_{1t} is nonzero only if the zero lower bound on interest rates is binding. The optimal state-contingent evolution of the endogenous variables $\{\pi_t, y_t, \hat{b}_t, \hat{\tau}_t\}$ is then characterized by these first-order conditions together with the constraints and the complementary slackness condition.

We ensure satisfaction of the tranversality condition for optimality by selecting the non-explosive solution to these equations, which is the one in which the zero bound ceases to bind after some finite number of periods (that depends on the random realization of the number of periods for which the natural rate of interest is negative), and hence the equilibrium corresponds after a finite number of periods to a steady state with zero inflation. We assume an initial public debt b_{t-0-1} equal to the constant value $\bar{b}$ in the steady state around which we approximate our objective and constraints,[23] so that we set $\hat{b}_{t_0-1} = 0$. Finally, we specify the initial lagged Lagrange multipliers to be those that would have been associated with an optimal commitment chosen prior to date t_0, assuming that at that time the occurrence of the decline in the natural rate of interest was assigned a negligible probability (though an optimal commitment was made about what should happen if the low-probability event were to occur). In a previously anticipated optimal steady state with a constant public debt level $\hat{b}_t = 0$, the Lagrange multipliers would have constant values equal to zero; hence we set $\varphi_{1,t_0-1} = \varphi_{2,t_0-1} = 0$.

We again assume the numerical parameter values given in the first column of Table 1, and again consider the effects of a real disturbance of the same kind as in Figure 1. Figures 2-4 display the optimal responses to this kind of disturbance. The solution to the first-order conditions characterizing optimal policy is of the following form. For the first T_1 quarters (where T_1 is random and equal at least to 1), $r_t^n = \underline{r} < 0$, and the zero bound is binding. Hence (1.27) holds with equality, with the value $\underline{r}$ substituted for r_t^n each period, and $\varphi_{1t} > 0$. In quarter T_1 and thereafter, $r_t^n = \bar{r} > 0$. But in quarters T_1 through $T_2 - 1$, the zero bound continues to bind, so that (1.27) holds with equality, now substituting the value $\bar{r}$ for r_t^n. From quarter T_2 onward (where the value of $T_2 \geq T_1$ is known with certainty once T_1 is

[23] Here that value is $\bar{b} = 0$, but the same comment applies to the way in which we set $\hat{b}_{t-0-1}$ in our later example with $\bar{b} > 0$.

realized), the zero bound ceases to bind. For these periods, we solve the system of equations in which the requirement that (1.26) hold with equality is replaced by the requirement that (2.5) hold with equality.

Because the dynamics from quarter T_1 onward are completely deterministic, (2.4) requires that φ_{3t} be constant for all $t \geq T_1$. Condition (2.3) then requires that φ_{2t} be constant for all $t \geq T_1$ as well. Since $\varphi_{1t} = 0$ for all $t \geq T_2$, it then follows from (2.1) that $\pi_t = 0$ for all $t \geq T_2 + 1$, and from (2.2) that y_t take some constant value for all $t \geq T_2 + 1$. It then follows from (1.19) that $\hat{\tau}_t$ is constant for all $t \geq T_2 + 1$, and from (1.32) that $\hat{b}_t$ is constant for all $t \geq T_2 + 1$. Thus for all $t \geq T_2 + 1$, the economy is again in a zero-inflation steady state, possibly involving different long-run values of $\hat{b}_t, \hat{\tau}_t$, and y_t than in the initial steady state.

Figures 2 and 3 plot the state-contingent paths of inflation, the output gap, the nominal interest rate, the tax rate, and the level of public debt in this solution, for each of the possible realizations of T_1. (As in Figure 1, these figures superimpose the paths for $T_1 = 1, T_1 = 2$, and so on.) To clarify what happens under a typical contingency, Figure 4 shows the paths for the nominal interest rate, the output gap, and inflation in the case that $T_1 = 10$ quarters. (In this case, $T_2 = 14$ quarters.) The first panel of Figure 4 also plots the path of the natural rate of interest (the dotted line), showing that it falls to the level $\underline{r}$ in quarter zero, remains there for 10 quarters, and returns to the steady-state level $\bar{r}$ again in quarter 10.

Several features of the optimal policy are worthy of comment. First of all, while it would be possible for policy to restore the economy to an optimal steady state from quarter T_1 onward — this would involve zero inflation and maintaining a constant level of public debt, at whatever level of would have been accumulated by that date — and while there are no disturbances from that date onward to justify a non-stationary policy, an optimal policy involves a commitment *not* to behave in this way. Thus optimal policy is *history-dependent*, as in the analysis of Eggertsson and Woodford (2003). Here we see that when both monetary and fiscal policy are chosen optimally, *both* are history-dependent: the inflation rate, the nominal interest rate, and the tax rate all temporarily take values different from what their eventual long-run values will be, that depend on the duration and severity of the previous decline in the natural rate of interest.

The way in which optimal monetary policy is history-dependent is again similar to the conclusions obtained in Eggertsson and Woodford (2003). Optimal policy involves a commitment to keep nominal interest rates low for a period of time after the natural rate returns to its normal level; for example, in the case that $T_1 = 10$, the natural rate returns to its normal level in quarter 10, but optimal policy maintains a zero interest rate for three more quarters (quarters 10-12), and a nominal interest rate far below the natural rate in quarter 13 as well. Monetary policy remains loose for several more quarters despite a strong output boom in quarter 10 and continued inflation; however, both the price level and the output gap are stabilized a few quarters later.

Our results do, however, show that there is also a role for tax policy in responding to a liquidity trap, even if it would not be desirable to vary tax rates in response to cyclical disturbances that are not severe enough to cause the zero lower bound to bind. In our present model, the optimal response to the kind of disturbance considered would involve no change in tax rates (as well as no change in the inflation target) as long as $\underline{r}$ were non-negative. Instead, in the case of a disturbance large enough to cause the zero bound to bind, we find that it is optimal for tax policy to respond to the shock, as proposed by Feldstein (2002). But the optimal fiscal response is quite different from the conventional wisdom: we find that an optimal policy involves *raising* tax rates during the liquidity trap, while committing to *cut* them once the natural rate of interest is positive again, even though the latter commitment requires taxes to be cut during an inflationary boom.

Our unconventional conclusions regarding optimal tax policy are actually a fairly straightforward consequence of the role of taxes in our model. The tax rate matters for inflation and output determination (and hence, for those variables that appear in the loss function (1.16)) only through its effect on the aggregate-supply relation (1.19): a higher tax rate increases the real marginal cost of supply for any given level of output, and so increases the inflation rate associated with a given level of output and given inflation expectations. The value of departures from complete tax smoothing depends on the value of being able

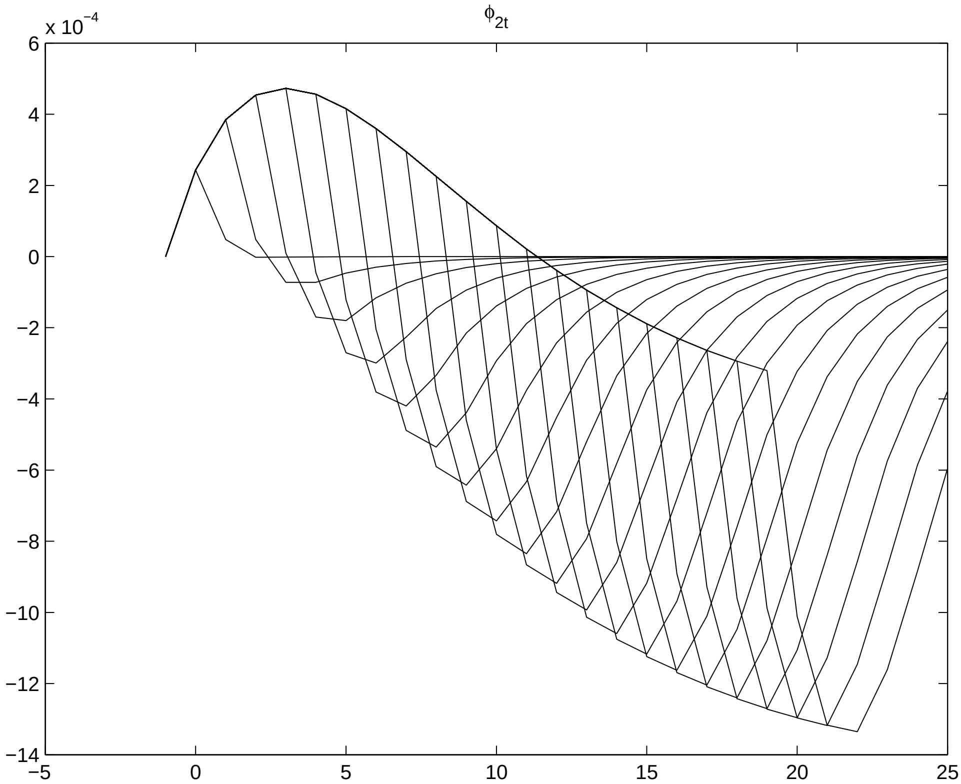

Figure 5: The evolution of φ_{2t} in the equilibrium of Eggertsson and Woodford (2003).

to shift the location of this aggregate-supply relation. In Eggertsson and Woodford (2003), we consider optimal monetary policy under the assumption that there is no policy instrument that can shift this constraint — which is equivalent to assuming that constraint (1.19) applies, but that the tax rate cannot be changed — and that constraint (1.27) is shifted by the disturbance to the natural rate of interest in exactly the same way as is assumed here. Thus the optimal state-contingent paths of inflation, output and interest rates in that paper represent a constrained-optimal solution to the problem considered here, where the constraint is that tax rates remain at their initial level. (Since this path of taxes is consistent with constraint (1.32), the problem corresponds to precisely the problem considered here, but under an additional constraint.)

We can then ask what the value to a planner would be, in the equilibrium computed in that paper, of shifting the aggregate-supply relation. This is indicated by the Lagrange multiplier associated with that constraint in the problem solved in Eggertsson and Woodford (2003). The state-contingent paths of the Lagrange multiplier φ_{2t} in that equilibrium, for each possible realization of T_1, are shown in Figure 5. We see that in the equilibrium computed in the earlier paper,[24] the value of φ_{2t} is positive during the period of the liquidity trap, and then negative later, before eventually approaching zero. This means that welfare would be improved if a positive cost-push shock were to occur during the time that the natural rate of interest is negative, followed by a negative cost-push shock after the natural rate becomes positive again. Once it is possible to use the VAT rate for stabilization purposes, an instrument is available that can be used to shift the constraint in both of these ways, subject only to the constraint that any changes in the path of the tax rate still satisfy the intertemporal government solvency condition (1.32). The fact that there is only a present-value constraint on the path of the tax rate implies that the shadow value φ_{2t} of shifting the AS relation should be smoothed over time; relative to Figure 5, this would mean lowering it during the period that the natural rate of interest is negative, and raising it in the early period following the return of the natural rate to its normal level. This direction of adjustment of the shadow value of further shifts in the AS relation requires that taxes be raised initially, and in exchange be lowered later.

2.2 Optimal Policy with Additional Fiscal Instruments

In the baseline case just considered, we assume that only a single type of public debt can be issued (riskless nominal one-period Treasury bills), and that only a single tax rate can be varied (a proportional sales tax such as a VAT). There is then a unidimensional fiscal policy decision to make each period: the fiscal authority must choose the tax rate, which implies a certain level of tax revenues and hence a certain quantity of Treasury bills in circulation. In reality, fiscal policy can be varied along multiple dimensions, because of the existence both of multiple taxes and of multiple forms of government debt. Here we consider the extent to

[24]The values plotted here are for the solution to our previous model using the parameter values reported in the first column of Table 1.

which improved stabilization would be possible using additional fiscal instruments, and the extent to which this might eliminate the need for history-dependent monetary policy.

We first consider the consequences of multiple forms of public debt. In the most favorable case, suppose that the government can issue state-contingent securities of an arbitrary form. In this case, b_{t-1} is no longer a single quantity determined at date $t-1$, but may instead take a value that depends on the exogenous disturbances realized at date t.[25] The intertemporal government solvency condition is then a single constraint, rather than a separate constraint for each state that may be realized at any date t. Condition (2.4) then becomes

$$\varphi_{3,t+1} = \varphi_{3t}$$

for each possible state at date $t + 1$; the Lagrange multiplier associated with the solvency condition never changes. Since this multiplier is zero in the steady state that is assumed to exist prior to the occurrence of the disturbance to the natural rate of interest in period t_0, it follows that, under any optimal policy commitment entered into prior to date t_0, the value of b_{t_0-1} will vary with the disturbance at date t_0 in such a way that the multiplier continues to equal zero, and so $\varphi_{3t} = 0$ for all $t \geq t_0$. It follows from (2.3) that $\varphi_{2t} = 0$ as well.

Thus the optimal evolution of inflation and of the output gap are characterized by equations (2.1) and (2.2), with the φ_2 terms eliminated, together with (1.27), (2.5), and the complementary slackness relation between these last two inequalities. (We solve these equations under the initial condition $\varphi_{1,t_0-1} = 0$.) Condition (1.19) can then be solved for the implied evolution of the tax rate, and condition (1.20) for the implied evolution of the state-contingent value of government debt maturing each period.

The solution to these equations is exactly the one already shown in Figures 2-4, since the solution to our previous system of equations already involved $\varphi_{2t} = \varphi_{3t} = 0$ for all $t \geq t_0$. [Add further discussion.]

It follows that even if the government can issue state-contingent debt of arbitrary type, the optimal state-contingent evolution of inflation, the nominal interest rate, the tax rate,

[25] In this case, the notation with a subscript $t - 1$ is unappealing, but the variable enters the budget constraints in the same way as before, so we do not propose alternative notation.

and so on are exactly the same as in the previous section. Thus both optimal monetary policy and optimal tax policy continue to be history-dependent, as shown above. The same result will be obtained, of course, in the event that there are multiple forms of government debt, but not enough types to span all states of nature. For in fact the result just obtained shows that optimal policy, even in the event of fully state-contingent government debt, requires only that riskless one-period nominal debt be issued. The optimal paths of all of the variables shown in Figures 2-4 are therefore those shown in those figures regardless of the types of debt that can be issued by the government, as long as a riskless one-period nominal bill is *one* of the possible types.[26]

Similarly, our results in the previous section would be unchanged by allowing for the existence of alternative forms of taxes. As we have seen, it is already true in our baseline case that the intertemporal government solvency condition does not preclude a state-contingent evolution of the VAT tax rate of the kind that would be chosen in the absence of any such constraint (this is the meaning of the result that $\varphi_{3t} = 0$ at all times). As a consequence, the VAT tax rate can be used to shift the aggregate-supply relation in whichever way may be desired in each possible state of the world at each date. The AS relation is then no constraint on the possible state-contingent evolution of inflation and output (this is the meaning of the result that $\varphi_{2t} = 0$ at all times). Given this, there is no possibility of achieving a better outcome by varying the path of some other distorting tax (say, a tax on labor income) that also shifts the AS relation to some extent; nor is there a need for a tax that can achieve some different relation between the size of state-contingent shifts in the AS relation and the state-contingent changes in government revenues. In our baseline analysis, the only binding constraint on the achievable paths for inflation and output (the only two variables that matter for the stabilization objective) is (1.27), which binds until the random date T_2. Hence additional fiscal instruments are relevant only to the extent that they affect this

[26]This irrelevance result is special to the case of zero initial public debt, assumed here as in the previous section. However, the special case considered here suffices to show that the mere existence of a large range of types of government debt does not eliminate the need for a commitment to history-dependent monetary policy.

constraint, which the VAT does not.

The only additional fiscal instrument that would make a difference is thus one that could affect the desired timing of private expenditure for a given expected path of real interest rates. An example would be an American-style sales tax, that is paid by the consumer in addition to the posted (sticky) price, rather than being included in the posted price as with a VAT.[27] If we let τ_t^s be the proportional sales tax in period t, then the Euler equation (1.9) generalizes to

$$1 + i_t = \beta^{-1} \frac{\tilde{u}_c(Y_t - G_t; \xi_t)((1 + \tau_t^s)P_t)^{-1}}{E_t[\tilde{u}_c(Y_{t+1} - G_{t+1}; \xi_{t+1})((1 + \tau_{t+1}^s)P_{t+1})^{-1}]}. \tag{2.6}$$

It follows that (1.27) takes the more general form

$$y_t \leq E_t y_{t+1} + \sigma[r_t^n + E_t \pi_{t+1} + E_t(\hat{\tau}_{t+1}^s - \hat{\tau}_t^s)], \tag{2.7}$$

where $\hat{\tau}_t^s \equiv \log(1 + \tau_t^s/1 + \bar{\tau}^s)$.

We see that the effect of a decline in the natural rate of interest on the tightness of the constraint (2.7) can be offset by reducing the sales tax relative to its expected future value — either by temporarily lowering the tax rate, or by promising to raise it in the future.[28] In particular, one can completely offset the effects of any stochastic variation in the natural rate of interest by adjusting the sales tax rate (relative to its steady-state level) in proportion to variations in the infinite-duration natural rate of interest, according to the formula[29]

$$\hat{\tau}_t^s = \sum_{T=t}^{\infty} E_t[r_T^n - \bar{r}]. \tag{2.8}$$

[27] The two types of sales taxes have different consequences in our model only because of the stickiness of prices. When prices are sticky, it matters whether it is the pre-tax price or the price inclusive of tax that is sticky.

[28] This is essentially the proposal of Feldstein (2002), except that he argues that an expected increase in a VAT rate will have the desired effect, which we find not to be the case.

[29] Feldstein (2002) proposes instead a commitment to steadily raise the sales tax rate relative to its initial level, eventually reaching a permanently higher rate. Feldstein's proposal has the disadvantage that, in the event that the length of time that the natural rate is negative is stochastic, the eventual cumulative increase in the sales tax rate would also have to be stochastic, so that the eventual long-run tax rate is uncertain. Furthermore, if a similar disturbance were to occur several times over a sufficiently long period of time, and were to be dealt with each time in accordance with the Feldstein proposal, the sales tax would eventually reach a very high level. Under our proposal, instead, the sales tax rate is always expected to eventually return to a fixed long-run tax rate $\bar{\tau}^s$.

Under this rule for adjustment of the sales tax rate, zero inflation and a zero output gap at all times will be consistent with the Euler equation (1.24) as long as the nominal interest rate is always equal to $\bar{r} > 0$, a requirement that can never be in conflict with the zero lower bound.

The effects of the variations in an American-style sales tax required by (2.8) on the aggregate-supply relation can in turn be neutralized by exactly offsetting variations in the VAT rate, so that the amount of the tax that is included in the posted price is varied without any change in the total tax wedge. (Such a shift in tax policy has no effect on the flexible-price equilibrium level of output, and hence no effect on the aggregate-supply relation.[30]) A shift in tax policy of this kind is also revenue-neutral, and so will not imply any violation of the intertemporal government solvency constraint (1.20).[31]

Hence when both kinds of taxes exist and can be adjusted in a state-contingent fashion with sufficient flexibility, it is possible to achieve both zero inflation and a zero output gap at all times (and so a first-best outcome) even when the natural rate of interest is occasionally negative. In such a case, there is no need for monetary policy to be history-dependent — neither the inflation target nor the level of nominal interest rates should depend on the past history of disturbances (nor on current or anticipated future disturbances, for that matter). However, this is a case in which fiscal policy can be used to ensure that the zero bound never binds. It remains quite generally true that *if* the zero bound sometimes binds, an optimal monetary policy involves a commitment to looser policy than would be associated with a strict zero inflation target for a period of time after it again becomes feasible to target zero

[30]Auerbach and Obstfeld (2004) criticize the proposal of Feldstein (2002), on the ground that in their optimizing model a commitment to increase the sales tax rate over time while the economy is in a "liquidity trap" lowers welfare. But this reduction in welfare results from the adverse supply-side effects of a higher sales tax that is not offset by lowering some other tax, as proposed by Feldstein. Our analysis here shows that it is possible to achieve the effect on demand sought by Feldstein without any supply-side effect, through a coordinated change in the expected paths of two different taxes.

[31]The argument here relies on the non-existence of fiscal stress effects of the real disturbance that shifts the natural rate of interest. Since the case described in the text involves no variation in interest rates in response to the disturbance, there will be no fiscal stress effect, even in an economy with a positive initial debt, as long as the disturbance has no effect on the target level of output $\hat{Y}_t^*$. Even when $\hat{Y}_t^*$ is changed, the existence of a sufficient number of fiscal instruments may make it possible for a policy consistent with (2.8) to also have no effect on the aggregate-supply relation or on intertemporal government solvency.

inflation.

It is also unclear how realistic it is to suppose that a sales tax can be adjusted in the fashion required to eliminate the problem of the zero bound under all circumstances. Few countries have a sales tax of the particular type required to offset the effects on the intertemporal Euler equation of variations in the natural rate of interest. Even in the case of the U.S., sales taxes of this kind are imposed at the state or local level, but not by the federal government, so that variations in a sales tax rate is not a potential tool of national fiscal policy. Furthermore, sales taxes would have to be fairly large to be used in the way proposed above. In the case of the real disturbance considered in our numerical examples above, equation (2.8) requires that the sales tax be reduced by 15 percentage points when the real disturbance occurs that lowers the natural rate of interest to -2 percent. This would not be possible, supposing that the tax rate must be non-negative at all times, unless $\bar{\tau}^s$ is at least 15 percent. Hence a substantial portion of government revenues would have to be raised by the sales tax under normal circumstances; and even granting that, it would have to be possible to dramatically alter the tax rate temporarily in response to the real disturbance. Barring such favorable circumstances, it seems unlikely that tax policy can be used to fully undo the consequences of variation in the natural rate of interest, and hence to eliminate the need for history-dependent monetary policy.[32]

2.3 Optimal Policy with Public Debt and Tax Distortions

We now consider the robustness of our results to the consideration of a situation in which there exists a positive level of public debt, and accordingly a positive steady-state tax rate. In this (more realistic) case, we must take account of the first-order effects on the government budget of variations in inflation, interest rates, or the size of the real tax base. The intertemporal government solvency constraint accordingly takes the more general form (1.20)

[32] Other kinds of tax incentives might affect the timing of expenditure as well, such as enactment of an investment tax credit during the period that the natural rate of interest is negative. Here we have emphasized what can be achieved with a variable sales tax rate, both because it is simple to analyze in the context of our model, and because it allows us to discuss Feldstein's (2002) proposal.

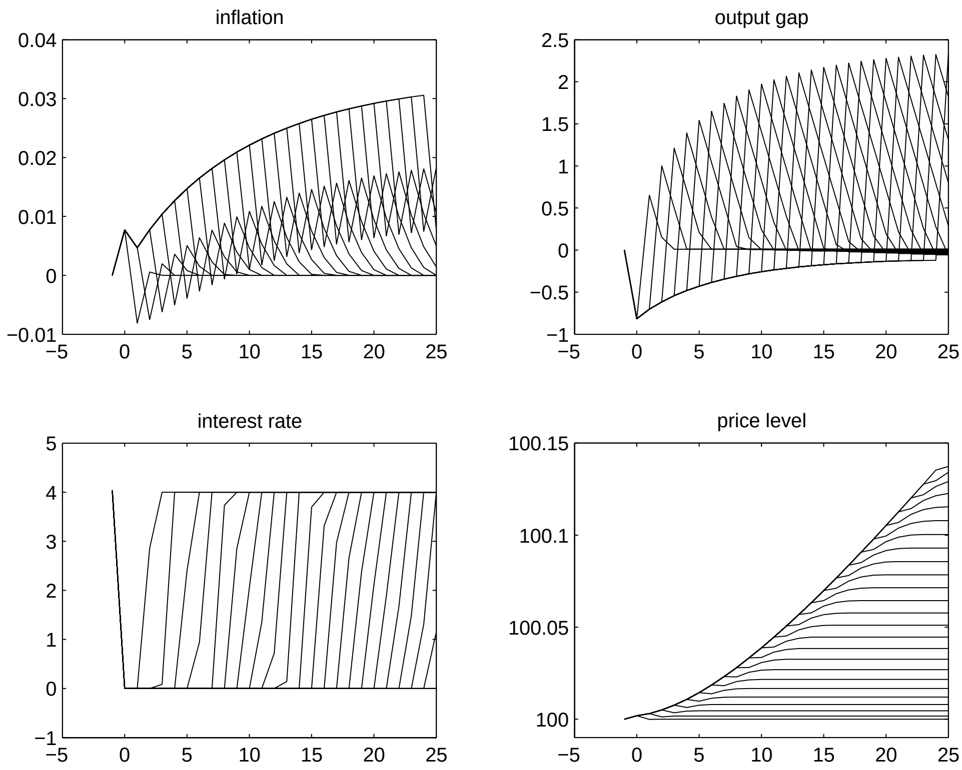

Figure 6: Optimal responses to a decline in the natural rate of interest: a case with positive debt but no fiscal stress effect.

rather than (1.32).

A similar Lagrangian method can be used in this more general case. The first-order conditions (2.1) – (2.2) must be replaced by

$$q_\pi \pi_t - \beta^{-1}\sigma\varphi_{1,t-1} + \varphi_{2t} - \varphi_{2,t-1} - s_b\varphi_{3t} + s_b\varphi_{3,t-1} = 0, \quad (2.9)$$

$$q_y y_t + \varphi_{1t} - \beta^{-1}\varphi_{1,t-1} - \kappa\varphi_{2t} - s_b(1 - \beta(1 - \sigma^{-1}))\varphi_{3t} + s_b\sigma^{-1}\varphi_{3,t-1} = 0, \quad (2.10)$$

while conditions (2.3) – (2.5) and the complementary slackness relation apply as before. Once again, we solve these conditions using initial conditions $\varphi_{j,t_0-1} = 0$ for $j = 1, 2, 3$.

As a numerical example with a level of public debt similar to that of a number of industrial economies, we now calibrate our model with the parameter values indicated in the second column of Table 1. The only changes are our assumption now of an initial public debt, and hence a steady-state public debt $\bar{b}$, equal to 60 percent of steady-state annual GDP, or

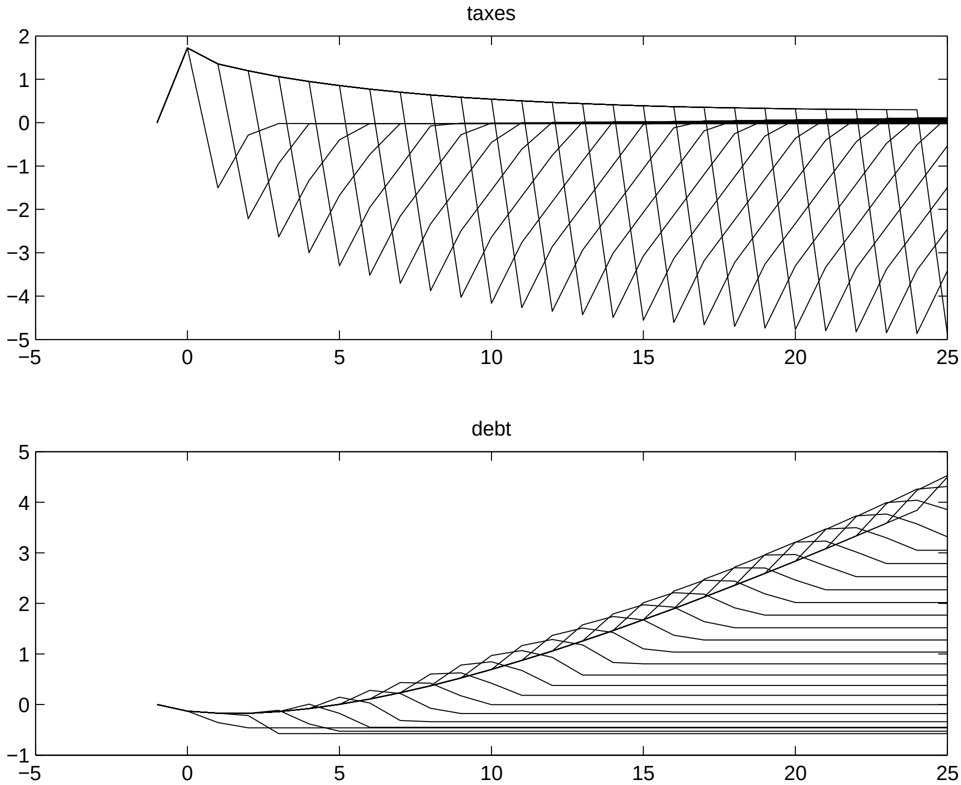

Figure 7: Optimal responses of fiscal variables: a case with positive debt but no fiscal stress effect.

2.4 quarters' GDP; and our assumption of steady-state government transfers of a size that require steady-state tax collections to equal 20 percent of national income,[33] along with the implied changes in other parameters that depend on the values of these. We consider the optimal response to the same kind of disturbance to the natural rate of interest as in Figures 2-4, and for the sake of simplicity we shall first consider a disturbance that has no fiscal stress effect, just as in the earlier figures.

The optimal responses under this variation in parameter values are shown in Figures 6 and 7, which have the same format as Figures 2 and 3. The optimal equilibrium in this case has many similarities to the one displayed in Figures 2 and 3 for the baseline model. Most

[33]This parameterization differs from the one used in Benigno and Woodford (2003) in that government spending is assumed to consist entirely of transfers rather than of purchases of goods and services by the government. Here we maintain the assumption that $s_G = 0$ even in our high-debt case, so that our real disturbances still have no cost-push effects, in order not to introduce a further complication into our discussion of optimal policy.

notably, the optimal policy again involves a commitment to create an inflationary boom (using both monetary and fiscal stimulus) in the period immediately following the return of the natural rate of interest to its normal level. And as in Figure 3, we again see in Figure 7 that it is optimal to raise the tax rate during the period in which the natural rate is low, while committing to lower it once the natural rate returns to its normal level.

Optimal policy for the high-debt economy differs quantitatively, however. In particular, the optimal degree of inflation that should be created in the periods in which the natural rate of interest is negative is greater in the case of a positive initial public debt. This indicates that the existence of a positive nominal public debt (and the availability only of distorting taxes as a source of government revenue) increases the desirability of responding to a decline in the natural rate of interest through an inflationary policy. Furthermore, this result does not reflect an incentive to inflate away the value of nominal public debt *ex post* that a policymaker would instead wish to commit itself in advance not to yield to. For the responses shown in Figures 6 and 7 represent the responses to a real disturbance that lowers the natural rate of interest under the continuation of an optimal state-contingent commitment that would have been chosen prior to quarter zero, and thus prior to learning that the disturbance would occur.

In the case of a positive steady-state debt level, it is still theoretically possible for a real disturbance to lower the natural rate of interest while having no effect on fiscal stress, as maintained in the last numerical example. However, as discussed in section 1.3, it is more plausible that a disturbance that could lower the natural rate of interest sufficiently for the zero bound to bind would also reduce fiscal stress. We accordingly consider an example of that more realistic kind. Specifically, we now assume that the real disturbance that lowers the natural rate of interest has no effect on the target level of output $\hat{Y}_t^*$. This implies that the fiscal stress is lowered to a value $\underline{f} < 0$ when the natural rate of interest falls to the level $\underline{r}$, and that $f_t = \underline{f}$ for as many quarters as $r_t^n = \underline{r}$, while $f_t = 0$ again once $r_t^n = \bar{r}$.[34]

In order to determine optimal policy in this case, we solve the same system of equations

[34] In our numerical example, $\underline{f}$ = -0.33.

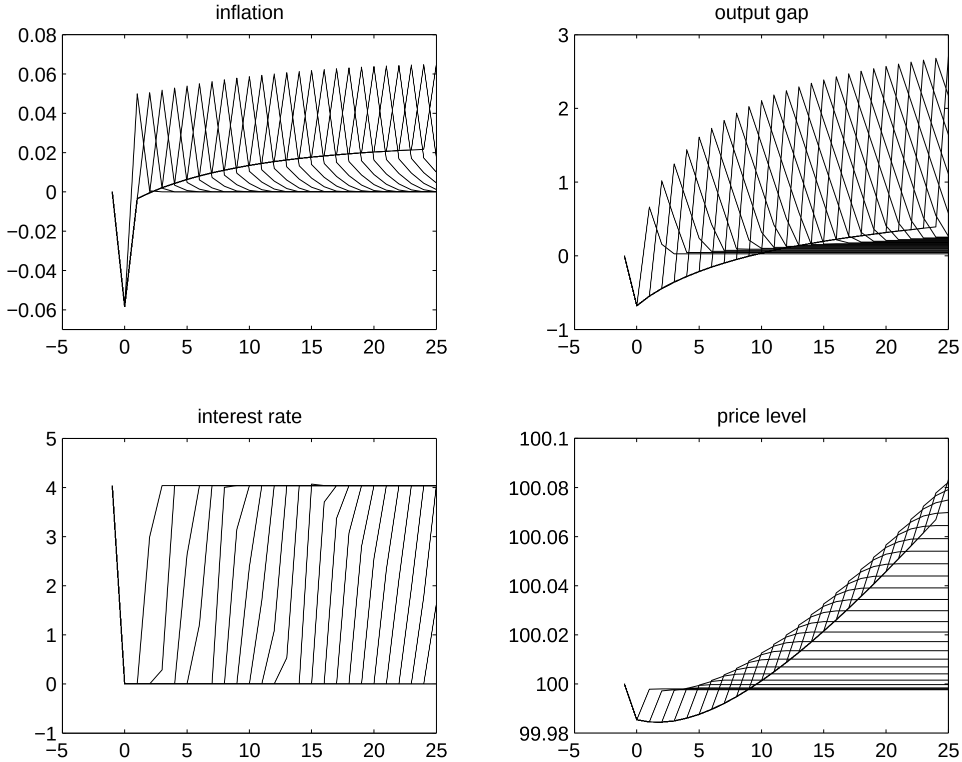

Figure 8: Optimal responses to a decline in the natural rate of interest: a case with positive debt and a negative fiscal stress effect.

(and with the same parameter values) as in the case of Figures 6-7, except that in equation (1.20) we now set $f_t = \underline{f}$ in periods zero through $T_1 - 1$. The optimal responses to this type of real disturbance are shown in Figures 8 and 9, which have the same format as Figures 6 and 7.

The fact that in this case the real disturbance also lowers fiscal stress has a number of consequences for the way that optimal policy responds to the shock. Not surprisingly, taxes do not need to be increased as much in this case, and it is possible to permanently lower both the tax rate and the level of public debt as a result of the disturbance. During the period over which the effects of the real disturbance on both the natural rate of interest and fiscal stress persist, inflation increases less and the output gap falls less, as a result of the smaller increase in the tax rate. But despite these quantitative differences, the general character of the optimal state-contingent evolution is similar to what we obtained in the

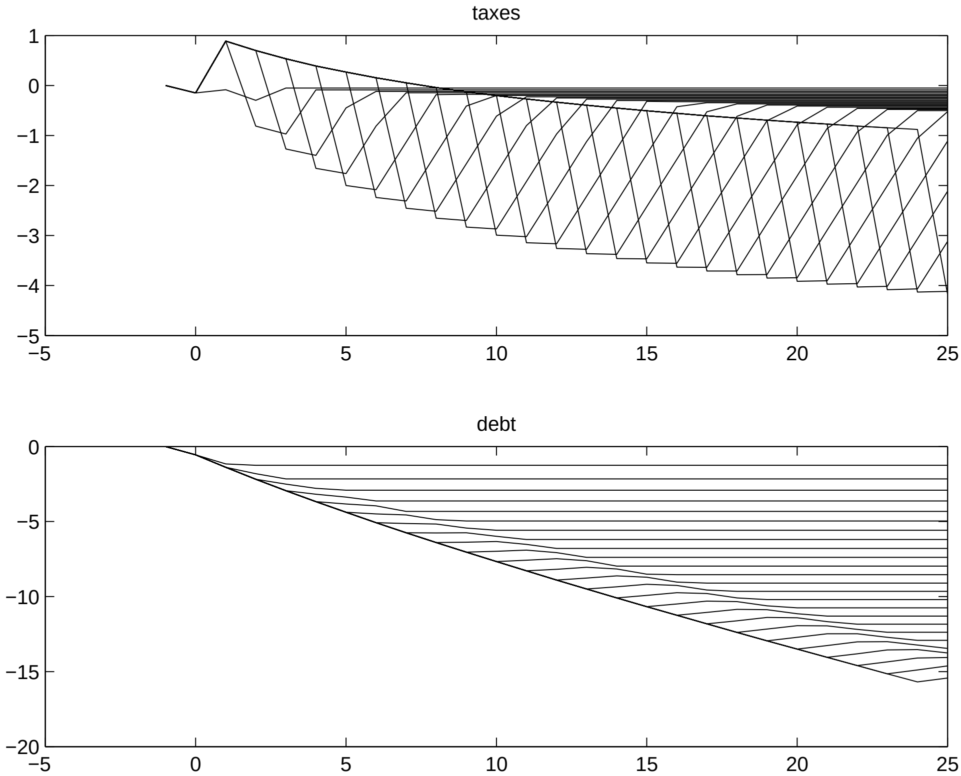

Figure 9: Optimal responses of fiscal variables: a case with positive debt and a negative fiscal stress effect.

baseline analysis.

Thus our main results in the baseline case continue to hold when we allow for a fairly substantial positive level of public debt and for fiscal stress effects of a plausible magnitude. We again find that tax policy should be adjusted in response to a real disturbance that causes the zero lower bound on interest rates to bind, even when the disturbance is of a type that would not justify any change in tax rates otherwise. But — at least when the available tax instrument affects the economy primarily through its supply-side effect on the marginal cost of supplying output — the optimal tax response is actually to raise taxes in the event of such a disturbance, while committing to lower them during the inflationary boom that monetary policy will also facilitate once the natural rate of interest returns to a normal level. And optimal monetary continues to be history-dependent in the same way as in the analysis of Eggertsson and Woodford (2003). The distortion created by the binding zero lower bound on interest rates should be mitigated through a commitment to use both monetary and fiscal policy to create a temporary output boom and rise in prices following the end of the real disturbance, though the price level should be stabilized again fairly soon. Such a policy will involve keeping nominal interest rates lower for several quarters than the level that would be required to maintain price stability.

3 The Cost of Purely Forward-Looking Policy

Our analysis of an optimal policy commitment in the previous section has shown that it would be history-dependent. But how much does history-dependence of policy actually matter? A purely forward-looking approach to the conduct of policy, which would suffice in the event that the zero bound does not bind,[35] will clearly entail some additional distortions relative to the optimal policy characterized above in the event that the zero bound is temporarily binding. Here we consider the nature of these distortions, and how much they matter quantitatively.

3.1 Optimal Forward-Looking Policies

We begin by considering the best possible policy rules to which the monetary and fiscal authorities might commit themselves, subject to the restriction that both monetary and fiscal policy be purely forward-looking.[36] By purely forward-looking policy we mean a rule of conduct which takes account only of the possible future evolution of the target variables (inflation and the output gap) given current conditions, and hence is independent of any past conditions that no longer affect inflation or output determination (Woodford, 2000).

In the case of the policy problem considered here, we observe that optimal policy, as

[35] Here we assume, as in our baseline case, that the real disturbances under consideration have no cost-push effects and no fiscal stress effects.

[36] While discretionary fiscal policy is purely forward-looking, if we consider only the case of a Markov perfect equilibrium, we do not here assume that the policymakers must be discretionary optimizers. Thus we allow commitment to policy rules that may be superior to the conduct of policy by discretionary optimizers, in order to identify the advantages of history-dependent policy rather than the benefits of commitment as such.

characterized in section 2.1,[37] has the property that the variables π_t,

$$\begin{aligned}\tilde{y}_t &\equiv y_t + (1-\beta)\psi\hat{b}_{t-1},\\ \tilde{\tau}_t &\equiv \hat{\tau}_t - (1-\beta)\hat{b}_{t-1},\\ \Delta_t &\equiv \hat{b}_t - \hat{b}_{t-1}\end{aligned}$$

each evolve in response to exogenous disturbances in a way that is independent of the level of the public debt (though the absolute levels of both the tax rate and the output gap depend on the level of the debt). That is, a unit increase in the initial public debt results, under an optimal policy commitment, in a permanently higher public debt by that amount, a permanently higher tax rate by the amount necessary to pay the interest on the additional public debt, a permanently lower level of output by the amount by which the flexible-price equilibrium level of output is reduced by the higher taxes, and no changes otherwise.

This shows that the optimal policy problem is actually independent of the level of the public debt, once written in terms of these transformed variables. Hence it makes sense to further require, in order for a policy to count as purely forward-looking, that the paths determined for these transformed variables should depend only on conditions that affect their possible evolution from the current date onward — which is to say, that the paths chosen for the transformed variables should be independent of the level of public debt.

In terms of the transformed variables, (1.19) becomes

$$\pi_t = \kappa[\tilde{y}_t + \psi(\tilde{\tau}_t - \hat{\tau}_t^*)] + \beta E_t \pi_{t+1}, \tag{3.1}$$

(1.21) becomes

$$\Delta_t = -\beta^{-1}\tilde{\tau}_t, \tag{3.2}$$

and (1.27) becomes

$$\tilde{y}_t \leq E_t y_{t+1} - (1-\beta)\psi\Delta_t + \sigma(r_t^n + E_t\pi_{t+1}). \tag{3.3}$$

[37]For the sake of simplicity, the discussion here is conducted under the same simplifying assumptions as in section 2.1, though a similar definition of the optimal purely forward-looking policy would be possible under more general assumptions.

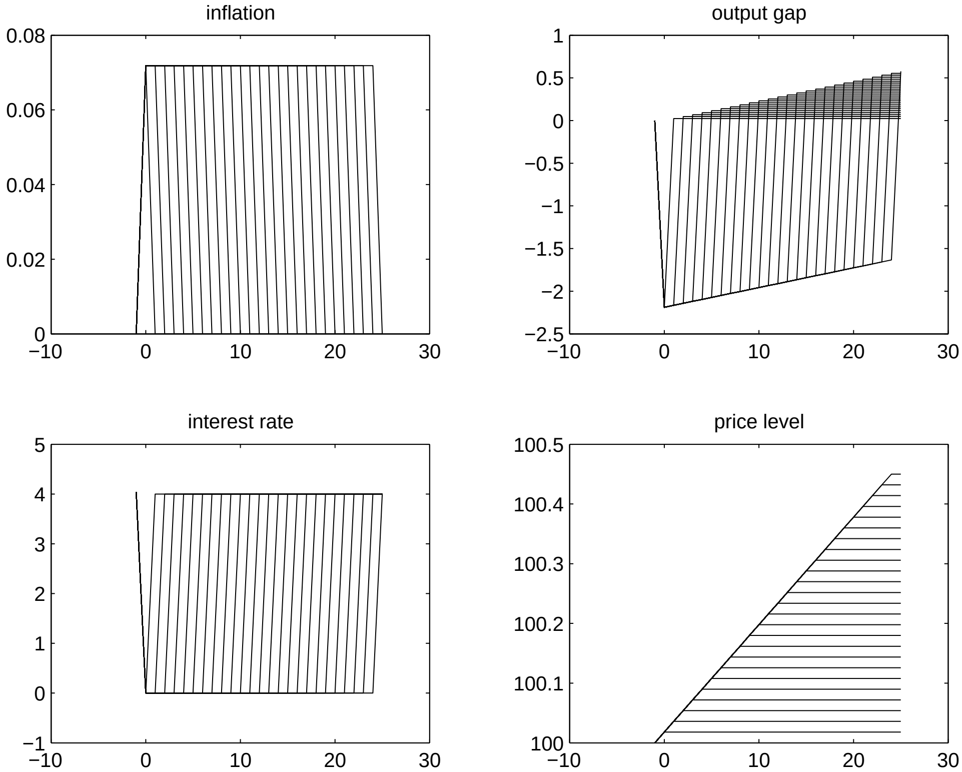

Figure 10: Responses to a decline in the natural rate of interest under optimal purely forward-looking policies.

Thus the possible state-contingent evolutions $\{\pi_t, \tilde{y}_t, \tilde{\tau}_t, \Delta_t\}$ from some date t onward depend only on the evolution of the process $\{r_t^n\}$ from that date onward. In the case that the natural rate of interest follows a Markov process, as assumed here, the only aspect of the economy's state at date t that is relevant for the possible equilibrium values of the transformed variables at that date or later is the current value r_t^n. We therefore define a purely forward-looking policy as one under which $\pi_t, \tilde{y}_t, \tilde{\tau}_t$, and Δ_t each period will depend only on the value of r_t^n in that period. (It then follows that i_t will depend only on the current value of r_t^n as well.)

In the case of the specific Markov process for the natural rate of interest considered in the numerical examples of section 2, a purely forward-looking policy can be fully specified by 8 numerical values, the values $(\bar{\pi}, \bar{y}, \bar{\tau}, \bar{\Delta})$ for the transformed variables whenever $r_t^n = \bar{r}$, and the values $(\underline{\pi}, \underline{y}, \underline{\tau}, \underline{\Delta})$ whenever $r_t^n = \underline{r}$. An optimal purely forward-looking policy is one in which these coefficients have been chosen so as to minimize the expected losses (1.16).

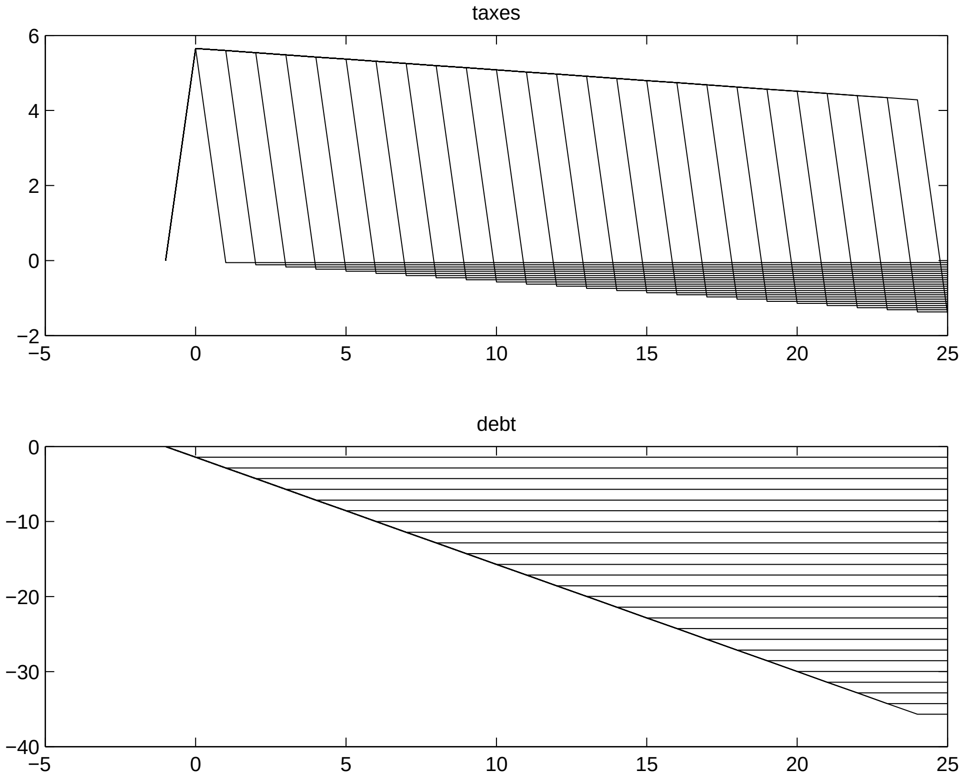

Figure 11: Responses of fiscal variables under optimal purely forward-looking policies.

We shall suppose furthermore that at the time that this optimal commitment is chosen, the occurrence of the state in which $r_t^n = \underline{r}$ is regarded as highly unlikely. It then follows that the policy commitment for the normal state should be chosen so as to maximize steady-state welfare, *i.e.*, to minimize $q_\pi \bar{\pi}^2 + q_y \bar{y}^2$. Hence the optimal forward-looking policy would involve a commitment to implement the optimal (zero-inflation) steady state as long as $r_t^n = \bar{r}$; thus $\bar{\pi} = \bar{y} = \bar{\tau} = \bar{\Delta} = 0$, and $\bar{i} = \bar{r}$.[38]

Given this solution, the policy commitment for the state in which $r_t^n = \underline{r}$ is chosen so as to minimize expected discounted losses conditional on being in that state, taking as given the commitment to pursue zero inflation as soon as the natural rate returns to its normal level. Under this stipulation, the policy commitments $(\underline{\pi}, \underline{y}, \underline{\tau}, \underline{\Delta})$ that are feasible are those

[38] The same conclusion about the conduct of policy once the natural rate returns to its normal level would also be reached under the assumption of discretionary optimization, independently of the probability that might have initially been assigned to the occurrence of the disturbance that temporarily lowers the natural rate of interest.

that satisfy the conditions

$$\underline{\pi} = \frac{\kappa}{1-\beta\rho}(\underline{y} + \psi\underline{\tau}), \tag{3.4}$$

$$\underline{\Delta}_t = -\beta^{-1}\underline{\tau}, \tag{3.5}$$

and

$$(1-\rho)\underline{y} \leq \underline{\Delta} + \sigma(\underline{r} + \rho\underline{\pi}), \tag{3.6}$$

implied by (3.1) – (3.3) respectively. These three conditions then identify the set of purely forward-looking policies.

Under the same assumptions, expected discounted losses looking forward from a date t_0 at which $r_t^n = \underline{r}$ are equal to

$$\begin{aligned} E_{t_0} \sum_{t=t_0}^{\infty} \beta^t \quad & \left[\frac{q_\pi}{2}\pi_t^2 + \frac{q_y}{2}y_t^2\right] \quad = \quad \frac{1}{1-\beta\rho}\left[\frac{q_\pi}{2}\underline{\pi}^2\right] \\ & + \quad \frac{q_y}{2}\sum_{j=0}^{\infty}\beta^j\rho^j(\underline{y} - j(1-\beta)\psi\underline{\Delta})^2 \quad + \quad \frac{q_y}{2}\sum_{j=1}^{\infty}\beta^j\rho^{j-1}\frac{1-\rho}{1-\beta}(j(1-\beta)\psi\underline{\Delta})^2 \\ = \quad & \frac{1}{1-\beta\rho}\left[\frac{q_\pi}{2}\underline{\pi}^2 + \frac{q_y}{2}\underline{y}^2\right] - \frac{\beta\rho}{(1-\beta\rho)^2}q_y(1-\beta)\psi\underline{y}\underline{\Delta} + \frac{\beta(1-\beta)(1-\beta^2\rho^2)}{(1-\beta\rho)^3}\frac{q_y}{2}\psi^2\underline{\Delta}^2 \\ \equiv \quad & L(\underline{\pi}, \underline{y}, \underline{\Delta}). \end{aligned}$$

The optimal purely forward-looking policy is then defined by the values $(\underline{\pi}, \underline{y}, \underline{\tau}, \underline{\Delta})$ that minimize $L(\underline{\pi}, \underline{y}, \underline{\Delta})$ subject to constraints (3.4) – (3.6).

It is easily shown that the function $L(\underline{\pi}, \underline{y}, \underline{\Delta})$ reaches its global minimum at $\underline{\pi} = \underline{y} = \underline{\Delta} = 0$. These values also satisfy (3.4) and (3.5), and they satisfy (3.6) as well if and only if $\underline{r} \geq 0$. Hence if $\underline{r} \geq 0$, the optimal purely forward-looking policy involves a constant zero inflation rate and a constant tax rate, regardless of the fluctuations in the natural rate of interest, and a nominal interest rate that perfectly tracks the natural rate of interest. Instead, if $\underline{r} < 0$, the zero lower bound binds in the low-natural-rate state.

In our baseline numerical example, considered in section 2.1 above, the optimal purely forward-looking policy would lead to responses of the form shown in Figures 10 and 11.[39]

[39] In this example, the optimal forward-looking policy involves $\underline{\pi} = .07$ percentage points per annum, $\underline{y} = -2.2$ percentage points, $\underline{\tau} = 5.7$ percentage points, and $\underline{\Delta} = -5.7$ percentage points per quarter.

Under an optimal forward-looking policy, the tax rate would be raised while the economy is in the "liquidity trap," with the consequence that tax rates are then permanently lower once the economy exits from the trap, as a result of the reduction in the public debt. This is once more contrary to conventional wisdom, but in conformity with our results above regarding optimal fiscal policy when policy is not constrained to be purely forward-looking. As in the case of fully optimal policy, the optimal forward-looking regime actually creates a slight degree of inflation during the period that the zero bound is binding.

Even under the constraint that policy be purely forward-looking, we see that it is possible to improve greatly upon the outcome with pure tax smoothing, the case shown in Figure 1. (Note that the monetary policy assumed in Figure 1, a strict zero inflation target, is the optimal purely forward-looking monetary policy, given tax smoothing, since the zero lower bound does not allow any more inflationary policy in the periods when the natural rate of interest is negative.) An appropriate adjustment of the tax rate in response to the real disturbance — specifically, *raising* taxes while the natural rate of interest is negative, so that lower taxes will be anticipated once the natural rate returns to its normal level — can greatly improve the stabilization of both inflation and the output gap.

It may be surprising that the use of fiscal policy for stabilization purposes can have such a strong effect even when it is constrained to be purely forward-looking, whereas the optimal purely forward-looking monetary is unable to prevent severe deflation and an output collapse. The reason this is possible is that the size of the public debt is a state variable that naturally conditions future policy, and hence current fiscal policy can be used, in effect, to commit future policy to be more expansionary, even when future policy will be purely forward-looking. Our result here is reminiscent of the finding of Eggertsson (2004a) that (Markov-perfect) discretionary policy does not lead to nearly so bad an outcome when monetary and fiscal policy are *both* chosen by an optimizing policy authority as when monetary policy is chosen under discretion while fiscal policy is not used for stabilization purposes; in that analysis (based on a different model of tax distortions), fiscal policy is effective *only* because it provides a way in which current policy can be used to influence the expected future conduct of policy.

However, the way in which fiscal policy should be used to influence expectations regarding future policy is different in the present case; because future policy is not expected to be determined under discretion, there is no possibility of creating expectations of inflation following the return of the natural rate to its normal level by increasing the size of the nominal debt. Instead, the size of the public debt carried into the future is expected to influence the future level of taxes, that will in turn influence real activity and real incomes for supply-side reasons. But the expectation of a future output boom has important consequences for pricing and spending decisions while the economy is in the liquidity trap, allowing a considerable improvement in stabilization.

The tax increases while the economy is in the liquidity trap also help more directly, by providing a cost-push incentive for prices not to be cut. But here as well, it is not the immediate effect of the increase in supply costs that is beneficial, but rather the expectation that supply costs will remain high in future periods, if the disturbance continues, so that there is no ground for deflationary expectations; hence it is the commitment to keep taxes high in future periods, if the disturbance continues, rather than the current level of taxes as such, that is crucial to the effectiveness of the policy. Thus clear communication with the public about the way current conditions affect the outlook for future policy continues to be an essential element of effective policy, just as in the discussion of optimal monetary policy in Eggertsson and Woodford (2003).

Our results here suggest that Krugman's (1998) emphasis on the importance of creating expectations of inflation following the economy's exit from the "trap" is somewhat exaggerated. Mitigating the distortions created by the zero lower bound on interest rates (or for that matter, some other, possibly higher, lower bound) does depend critically upon the creation of expectations of a more expansionary policy in the future as a consequence of the current disturbance. However, expectations of a supply-side boom (while monetary policy is used to maintain stable prices) can also reduce distortions resulting from the trap to a considerable extent. Expectations of high future real incomes will reduce the incentives for either price cuts or low spending during the period of the trap. And once it ceases to be expected that prices will decline sharply while the zero bound binds, real interest rates will no longer be perceived to be so high relative to the current natural rate of interest; it is this mechanism that makes changes in expectations regarding future policy such a potent tool in our model.[40]

3.2 Optimal Fiscal Policy but a Strict Inflation Target

Finally, we consider the costs associated with the pursuit of a forward-looking monetary policy, in the case that an optimal, history-dependent fiscal policy is allowed. In this way we consider the best possible outcome under a monetary policy of this kind. In particular, we allow fiscal policy to be chosen so as to mitigate the consequences of the simple monetary policy rule.

Specifically, we shall suppose that monetary policy is described by a strict zero inflation target, that is pursued to the extent that the zero lower bound on nominal interest rates allows the target to be attained. Thus we again impose the requirement that (1.34) holds each period, along with (1.27), and that at least one of these holds with equality in any period. We note that this rule for monetary policy would be optimal in the event of disturbances small enough for the zero lower bound to be consistent with zero inflation; and that it is also the monetary policy pursued under an optimal purely forward-looking regime, as characterized in the preceding section. We now consider the consequences of choosing fiscal policy optimally (and in a history-dependent way) under the constraint that monetary policy will be of this kind.

Optimization subject to the additional constraint (1.34) requires only a small modification of the approach discussed in section 2.1. Our numerical results in the case of our baseline parameter values are plotted in Figures 12 and 13. Since the optimal policy in the absence of this constraint never involves deflation, and positive inflation over the entire period for which

[40]For similar reasons, variations in the expected level of government purchases can have a substantial "multiplier" effect during a liquidity trap, as shown by Eggertsson (2004b).

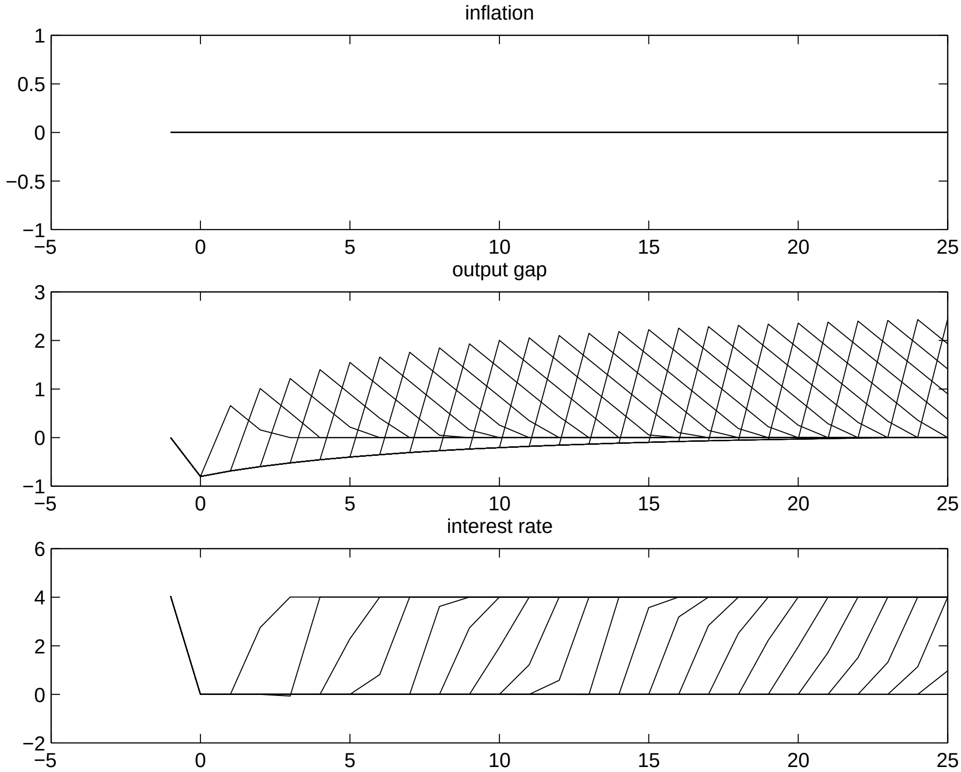

Figure 12: Responses to a decline in the natural rate of interest: optimal fiscal policy with a strict inflation target.

the zero bound is binding (see the first panel of Figure 2), it is perhaps not surprising that for our baseline parameter values, we find that the optimal policy subject to constraint (1.34) is one with zero inflation at all times. Fiscal policy is then used to achieve the minimum possible expected discounted sum of squared output gaps, consistent with the maintenance of zero inflation. Because of constraint (1.27), a zero output gap at all times is not possible; and a commitment to the creation of a positive output gap immediately after the natural rate of interest returns to its normal level reduces the extent to which this constraint requires the output gap to be negative while $r_t^n = \underline{r}$.

Thus optimal fiscal policy is history-dependent: it involves a commitment to cut taxes as soon as $r_t^n = \bar{r}$ again, so as to create an output boom even though monetary policy targets zero inflation. Taxes are instead raised while the natural rate of interest is negative, in order to build up the government assets that allow the commitment to later cut taxes to

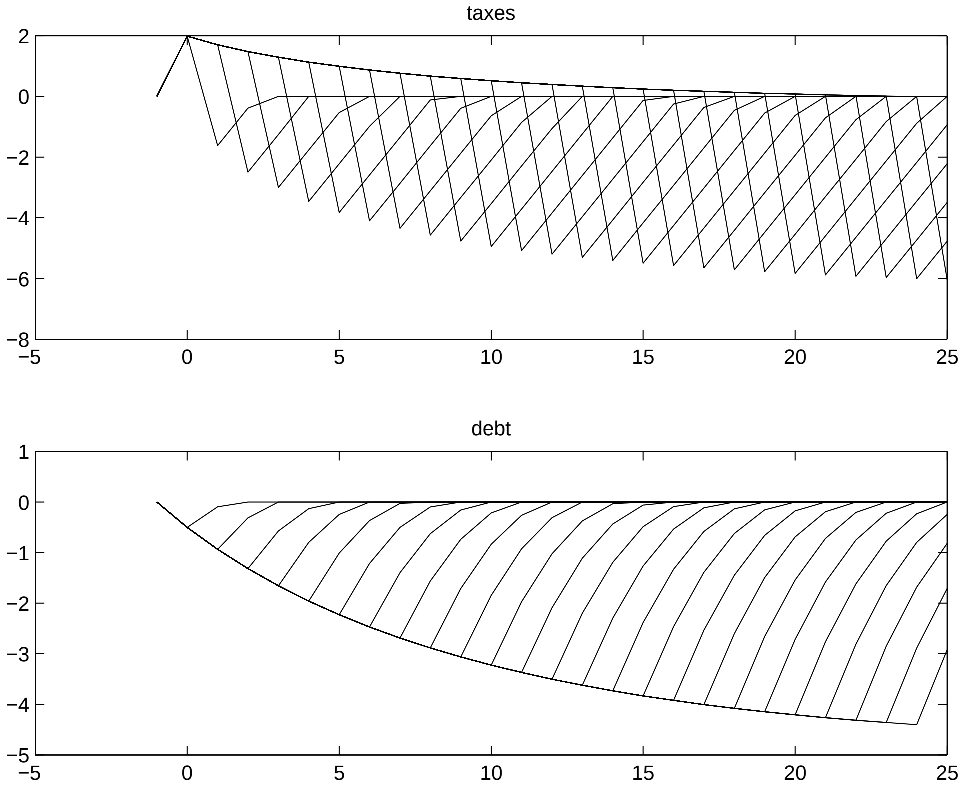

Figure 13: Optimal responses of fiscal variables in the case of a strict inflation target.

be consistent with intertemporal government solvency. The general character of the optimal fiscal commitment is similar to what we had obtained when both monetary and fiscal policy are optimally history-dependent (compare Figure 13 with Figure 3). However, the tax cuts that must be promised after the natural rate returns to its normal level are somewhat larger in the present case, as inflationary monetary policy cannot be used to help create the boom, the anticipation of which reduces the extent to which the zero bound requires a negative output gap in the earlier period. The extent to which the output gap can be stabilized through the use of history-dependent fiscal policy alone remains somewhat less than when both monetary and fiscal policy commitments are optimally chosen.

Table 2 summarizes our conclusions regarding the welfare consequences of alternative assumptions about the conduct of monetary and fiscal policy. (All values reported are for our baseline case with zero initial public debt.) In each case, the value of the loss function (1.16)

inflation target, tax smoothing	100.0000
optimal forward-looking policies	0.1333
optimal monetary, tax smoothing	0.0311
inflation target, optimal fiscal [VAT only]	0.0211
optimal monetary and fiscal [VAT only]	0.0208
inflation target, optimal fiscal [two taxes]	0
optimal monetary and fiscal [two taxes]	0

Table 2: Relative expected losses under alternative policies.

is reported, conditional upon the occurrence in period t_0 of the real disturbance that lowers the natural rate of interest to $\underline{r}$, under the rational-expectations equilibrium associated with the given policy regime. The losses are normalized so that 100 means the level of expected loss associated with the naive policy of strict inflation targeting combined with simple tax smoothing (discussed in section 1.4).

It is clear that even when only a VAT rate can be adjusted — or more generally, when the only available tax instruments are ones that have purely supply-side effects — an appropriate use of tax policy for stabilization purposes can be quite beneficial when the zero lower bound on interest rates binds. When only purely forward-looking policies are considered, the best fiscal rule of this type is a considerable improvement over the outcome with pure tax smoothing (the case shown in Figure 1): expected discounted losses fall by a factor of 750. In the case that monetary policy is assumed to adhere to a strict zero inflation target, an optimal fiscal commitment can do even better, cutting expected discounted losses by an additional factor of more than 6, so that losses are reduced to only 0.02 percent of what they would be with complete tax smoothing. With sufficient fiscal instruments — specifically, if it is possible to adjust both an American-style sales tax as well as a VAT rate with sufficient flexibility in response to real disturbances — even the remaining distortions can in principle be completely eliminated under a strict zero inflation target for monetary policy.

While we have found that except in the ideal case of the two taxes that are each adjusted optimally, optimal monetary policy continues to be characterized by the kind of history dependence stressed in Eggertsson and Woodford (2003), this sophisticated form of monetary policy commitment only matters greatly for welfare when fiscal policy is *not* used (or not properly used) for stabilization purposes. Even when only a VAT rate can be adjusted, if tax policy responds optimally to the real disturbance that makes the natural rate of interest temporarily negative, fiscal policy can eliminate most of the distortions associated with the "liquidity trap" scenario shown in Figure 1 even when monetary policy adheres to a strict zero inflation target. The further stabilization that can be obtained through an optimal monetary policy commitment only reduces expected discounted losses by an additional 1.3 percent. If fiscal policy can be adjusted in more flexible ways, the additional welfare gains from history-dependent monetary policy are even smaller (zero in the case of ideally flexible fiscal policy).

On the other hand, if fiscal policy does not respond to cyclical developments at all (the case of pure tax smoothing), a history-dependent monetary policy commitment can greatly improve stabilization, as shown in Eggertsson and Woodford (2003).[41] Hence the degree to which it is essential for a central bank to respond to a negative natural rate of interest by committing itself to a temporary loosening of policy later depends to an important extent on the degree to which it can expect fiscal policy to be adjusted in a way that serves stabilization purposes or not.

[41] Optimal monetary policy in the case of perfect tax smoothing results in an equilibrium of exactly the kind displayed in Figures 3-5 of Eggertsson and Woodford (2003), which we do not reproduce here, though we list the associated expected losses in Table 2.

References

Auerbach, Alan J., and Maurice Obstfeld, "The Case for Open-Market Purchases in a Liquidity Trap," NBER working paper no. 9814, July 2003.

—- —- and —- —-, "Monetary and Fiscal Remedies for Deflation," *American Economic Review Papers and Proceedings,* forthcoming, May 2004.

Benigno, Pierpaolo, and Michael Woodford, "Optimal Monetary and Fiscal Policy: A Linear-Quadratic Approach," *NBER Macroeconomics Annual 2003,* forthcoming.

Calvo, Guillermo A. (1983), "Staggered Prices in a Utility-Maximizing Framework," Journal of Monetary Economics, 12: 383-98.

Eggertsson, Gauti, "The Deflation Bias and Committing to be Irresponsible," chapter 2 of *Optimal Monetary and Fiscal Policy and the Liquidity Trap,* Ph.D. dissertation, Princeton University, June 2004.

—- —-, "Monetary and Fiscal Coordination in a Liquidity Trap," chapter 3 of *Optimal Monetary and Fiscal Policy and the Liquidity Trap,* Ph.D. dissertation, Princeton University, June 2004.

—- —- and Michael Woodford, "The Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity* 2003:1, pp. 139-211.

Feldstein, Martin, "Commentary: Is There a Role for Discretionary Fiscal Policy?" in *Rethinking Stabilization Policy,* Kansas City: Federal Reserve Bank of Kansas City, 2002.

Giannoni, Marc P., and Michael Woodford, "Optimal Interest-Rate Rules: I. General Theory," NBER working paper no. 9419, December 2002.

Krugman, Paul (1998), "It's Baaack! Japan's Slump and the return of the Liquidity Trap," Brookings Papers on Economic Activity 1998:2, pp. 137-187.

Svensson, Lars E.O., and Michael Woodford, "Implementing Optimal Policy through Inflation-Forecast Targeting," in B.S. Bernanke and M. Woodford, eds., *The Inflation Targeting Debate,* Chicago: University of Chicago Press, forthcoming 2004.

Woodford, Michael, "Commentary: How Should Monetary Policy be Conducted in an Era of Price Stability?" in sl New Challenges for Monetary Policy, Kansas City: Federal Reserve Bank of Kansas City, 1999.

—- —-, "Pitfalls of Forward-Looking Monetary Policy," *American Economic Review*

90(2): 100-104 (2000).

—- —-, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton: Princeton University Press, 2003.